"Multiscale reconstruction of shallow marine sediments using wavelet correlation"
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Introduction
The subject of studying surface waves is receiving new attention because of the potential in using surface waves for prediction of physical properties of near surface marine sediments. However, processing of surface waves normally relies on algorithms, which are do not allow sufficient discrimination between surface waves modes. This work extends and recasts the results of our previous studies on the wavelet cross-correlation analysis of surface waves. We are introducing multiscale cross-correlation in the spatial and time domains.

We applied the wavelet transform to seismic traces $\text{Wat}[\text{time}, \text{frequency}]$ and $\text{Wbt}[\text{time}, \text{frequency}]$, then calculated the cross-correlation function in the time domain $\langle \text{Wat}[\text{time}, \text{frequency}] \rangle$ and additionally carried out the cross correlation of wavelet fields in horizontal direction $X$ (distance along the interface) $\langle \text{Wgt}[\text{time}, \text{frequency}] \rangle$. The modulus and phases of multiscale cross-correlation function present the group and phase velocities at given frequencies and time delay with the spatial resolution defined as a minimum spatial lag $X$ (minimum distance between receivers). We present a comparison of using 1D and 2D multiscale cross-correlation techniques in terms of resolving the phase and group velocities of surface waves.

Continuous wavelet transform of function $f(t)$ at time $t$ relative to wavelet $\psi(t)$ kernel at frequency scale $f$:

$$\psi(f) = \int \frac{1}{\sqrt{f}} \phi \left( \frac{\tau}{f} \right) d\tau$$

Wavelet Cross-Correlation Function for $f(x)$ and $g(x)$:

$$WC_{xy} = \lim_{T \to \infty} \int_{-T}^{T} \int_{-T}^{T} Wf(x, \alpha) Wg(x + \beta, \alpha) \, dx \, d\alpha$$

We consider harmonic surface waves propagating on the ocean-sediment interface. The surface wave is a combination of propagational motion along the interface, where the phase is moving along horizontal coordinate, and syn-phase oscillations (modes) along $z$ where amplitude is rapidly decaying with depth. In the case of the depth-dependence shear modules, phase and group velocity of the surface waves perform frequency dependence - dispersion.

1D Wavelet Cross-Correlation for traces: 5 (2.5 km) and 3 (1.5 km)

Cross-correlation in time domain

$WCR_{\text{phase}} = \int \int [X(f, \tau)(\sqrt{f} d\tau + f^2)] = \int [X(f, \tau) \left( \frac{1}{\sqrt{f}} \frac{1}{2} \right)] (1 + |\tau|)$

$WCR_{\text{phase}}$ describes the phase shift difference between two signals at different locations for given time delay $\tau$. The change of phase $\theta$ is defined as change of phase velocities between time:

$V_1 = \frac{V_2}{V_1}$

and $V_2$ - phase velocities of surface waves recorded at different locations. When $V_1 = V_2$ - there are no changes in physical properties between receivers.

2D Wavelet Cross-Correlation function

Cross-correlation of the wavelet transformed seismic data in $X$ direction (distance along the interface) in addition to the time domain:

$$WC_{xy} = \lim_{T \to \infty} \int_{-T}^{T} \int_{-T}^{T} Wf(x, \alpha) Wg(x + \beta, \alpha) \, dx \, d\alpha$$

2D Wavelet Cross-Correlation for selected frequency range from 1.5 to 2.3 Hz

Conclusion: 2D spatial and time domains Cross-correlation of the wavelet transformed seismic traces extracts the information of coherent strength (moduli) and phase in terms of periods (frequencies), time delay and spatial shift and allows to monitor the changes of these parameters in both time and horizontal distance. The phase velocity dispersion can be studied directly from the phase field of 2D wavelet cross-correlation function. The peaks of wavelet correlation moduli perform the relative energy distribution in surface wave (and its modes) showing group velocity dispersion.