Influence of Temperature-Dependent Thermal Conductivity on Mantle Convection

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Abstract

Influence of variable thermal conductivity on thermal convection with temperature-dependent viscosity is studied by carrying out two-dimensional numerical experiments. I employed a two-dimensional rectangular box heated below and filled with a Boussinesq fluid. The infinite Prandtl number is imposed on the fluid, following a convention used in studies of mantle convection. The free-slip condition is imposed on the top and the bottom boundaries, and the reflective condition is imposed on the side boundaries. To simplify discussions, I separated the variable thermal conductivity into two components; depth-dependent thermal conductivity and temperature-dependent thermal conductivity.

First, I investigated thermal convection with the depth-dependent thermal conductivity and constant viscosity. Two- and three-layer models of thermal conductivity are adopted and I investigated influence of thermal conductivity on efficiency of heat transport. As thermal conductivities in both upper and lower thermal boundary layers increase, the Nusselt number increases. Increasing thermal conductivity in the interior causes more gentle decrease of the Nusselt number. The influence of the variation of thermal conductivity in the interior can be explained by an increase of the effective Rayleigh number. It is also found that horizontal scales of convective cells are longer as thermal conductivities in the thermal boundary layers are smaller than that in the interior. These features are reproduced by using an one-dimensional and analytical loop-model, suggesting the in-
interpretations on the basic physics discussed above.

Next, thermal convection with combination of temperature-dependent thermal conductivity and temperature-dependent viscosity is investigated. I found four convective regimes are dependent on viscosity contrasts between the top and the bottom in models with constant thermal conductivity; the small viscosity contrast (SVC), the transitional (TR-I, TR-II) and the stagnant-lid (ST) regimes characterized by degrees of the deformations in the upper thermal boundary layer. The SVC regime has no lid near the surface. The TR-I regime has a less stiff surface lid associated with cold plumes and the TR-II regime has a less stiff lid only in the lower part. The ST regime has a less stiff lid only in the lowermost part. The main factor producing the difference between these regimes is a vertical gradient of viscosity in the upper thermal boundary layer. The temperature-dependent thermal conductivity produces a viscosity profile with curvature in the thermal boundary layer, which produces a variation of a degree of diffusion on a convective flow in the upper thermal boundary layer. In a model with lattice thermal conductivity, a thinner part in which a convective flow is diffused rapidly appears around the bottom of the upper thermal boundary layer, but a thicker part in which a convective flow is not much diffused appears near the surface. Such an upper thermal boundary layer causes surface deformations reflecting the positions of stronger plumes. In the model with radiative thermal conductivity, however, a thicker part where a convective flow is not much diffused appears
around the bottom of the upper thermal boundary layer, but a thinner part where a convective flow is diffused rapidly appears near the surface. Such an upper thermal boundary layer causes little surface deformation which do not reflect a convective pattern.
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1 Introduction

Planetary heat transport produces driving forces for dynamics on planets and controls thermal histories of planets (Christensen, 1985). Main processes of planetary heat transport are thermal convection in mantles of planets. One of important problems on the planetary sciences is to recognize features characterizing mantle convection in each planet, for example, vigor of the mantle convection, efficiency of heat transfer for the mantle convection (Christensen, 1984) and dynamics caused by the mantle convection (Davies, 1986). The mantle convection can be assumed to be thermal convection heated from below. However, a convective structure of the mantle convection is more complex than that of the Rayleigh Benard Convection which is the simplest thermal convection heated from below. That is, because mantle rocks have temperature-dependent properties, for example, viscosity and thermal conductivity. Knowledge of thermal convection with these properties is important to understand features of the mantle convection.

To describe the mantle convection, conservation laws of mass, momentum and energy are adopted (McKenzie et al., 1974). These equations are expressed as,

\[ \frac{\partial u_i}{\partial x_i} = 0, \quad (1) \]

\[ 0 = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] + \rho_0 \alpha (T - T_0) g_i, \quad (2) \]

\[ \rho_0 c \frac{DT}{Dt} = \frac{\partial}{\partial x_i} \left( k \frac{\partial T}{\partial x_i} \right), \quad (3) \]

where \( u_i \) is the velocity, \( x_i \) is the coordinates, \( p \) is the pressure, \( \mu \) is the viscosity, \( \rho_0 \)
is the reference density, $\alpha$ is the thermal expansivity, $g_t$ is the gravity acceleration, $c$ is the specific heat, $T$ is the temperature and $k$ is the thermal conductivity. The Boussinesq approximation with the infinite Prandtl number is applied for eqs. (1), (2) and (3). The approximation that a fluid has the infinite Prandtl number is a convention in studies of the mantle convection.

$$Pr = \frac{\nu}{\kappa} = \infty,$$

where $Pr$ is the Prandtl number, $\nu = \mu/\rho_0$ is the kinematic viscosity and $\kappa = k/\rho_0 c$ is the thermal diffusivity. The kinematic viscosity of the mantle rocks ($10^{18} \sim 10^{20}$ m$^2$s$^{-1}$) is significantly larger than the thermal diffusivity ($10^{-6}$ m$^2$s$^{-1}$). So the Prandtl number is assumed to be infinite as usually used for the studies of the mantle convection.

Temperature-dependent viscosity of the mantle rocks decreases rapidly as temperature increases: a temperature change of 100 K causes a variation of the viscosity by a factor of 10 (Weertman, 1970). The viscosity with such a huge variation is included in eq.(2) describing a state of a convective flow, and the variation of the viscosity can directly affect convective structures of the mantle. Therefore, thermal convection with the temperature-dependent viscosity has been well studied by two-dimensional and three-dimensional numerical experiments and laboratory experiments (Nataf and Richter, 1982; Christensen, 1984; Davaille and Jaupart, 1993; Tackley, 1993; Solomatov, 1995). These previous works have shown that colder and stiffer upper thermal boundary layers appear, and degrees
of interactions between such boundary layers and underlying convective layers are dependent on viscosity contrasts across the layers. In these models, thermal conductivity has been assumed to be uniform all over the layers.

It has been known that thermal conductivity of the mantle rocks has also dependences on temperature (Birch and Clark, 1940; Schatz and Simmons, 1972; Hofmeister, 1999). However, a variation of the thermal conductivity across the mantle is not as large as the variation of the viscosity. In addition, the thermal conductivity is included not in eq.(2) but in eq.(3), and then, the variation of the thermal conductivity cannot affect a convective pattern directly. Because of such reasons, the variation of the thermal conductivity has not been included in the basic equations to describe a state of thermal convection. Therefore, I focus roles of the variable thermal conductivity on dynamics in planets. Under conditions that temperature is low relatively as expected to the lithosphere, for example, \( T < 1440 \) K for depth above 70 km (Hofmeister, 1999), a value of the thermal conductivity decreases as temperature increases. This contribution is called as the phonon conduction, or the lattice thermal conduction (Fujisawa et al., 1968; Kanamori et al., 1968; Kieffer, 1976). The lattice conduction is visualized as vibrations of moleculars (phonons) propagating along interatomic bonds. So, the lattice thermal conductivity is dependent on pressure: the lattice thermal conductivity becomes higher as pressure increases. On the other hand, a value of thermal conductivity increases as temperature increases under situations with high temperature as expected to the lower mantle, for example, \( T > 1870 \) K for
depth below 670 km (Hofmeister, 1999). This is the contribution of the radiation, or the radiative thermal conduction (Clark, 1957; Fukao et al., 1968). The radiation is also called as the photon conduction. These heat transfer processes are independent. So, the thermal conductivity in eq.(3) is expressed as:

\[ k = k_{\text{lat}} + k_{\text{rad}}, \]  

where \( k_{\text{lat}} \) is the lattice thermal conductivity, and \( k_{\text{rad}} \) is the radiative thermal conductivity (Schatz and Simmons, 1972).

A latest model of the thermal conductivity for the whole mantle is proposed by Hofmeister (1999). In her model, the thermal conductivity includes a larger contribution of the lattice thermal conductivity and a smaller contribution of the radiative thermal conductivity. Such a thermal conductivity produces vertical distribution of the thermal conductivity with a larger scale, which reflects a vertical profile of horizontally averaged temperature (see, Fig.1). In the lithosphere, the thermal conductivity decreases rapidly due to the contribution of the lattice thermal conductivity as the depth increases. In the athenosphere and the lower mantle, the thermal conductivity increases more gentle as the depth increases, because of pressure-dependence of the lattice thermal conductivity and temperature-dependence of the radiative thermal conductivity. In the lower thermal boundary layer called as D” layer, the thermal conductivity decreases rapidly because of high temperature and the lattice thermal conductivity. Such a vertical profile with a larger scale is important for consideration of averaged features char-
acterizing the mantle convection, for example, efficiency of heat transfer. It is a very important knowledge for studying thermal histories of planets using parameterized convection models (Sharpe and Peltier, 1978). In section 2, influence caused by such a vertical variation of the thermal conductivity is investigated.

The thermal conductivity also produces both vertical and horizontal variations with smaller scales on the large scale distribution demonstrated above. Such smaller variations can vary local distribution of temperature, which can also vary viscosity distribution with smaller scales. That is, such structures with small scales are important for examining local deformations in the mantle convection with the temperature-dependent viscosity. Therefore, I investigate influence of the temperature-dependent thermal conductivity on thermal convection with the temperature-dependent viscosity in section 3.

Finally, I summarize the influence caused by the temperature-dependent thermal conductivity on thermal convection in section 4.
2 Influence of Depth-Dependent Thermal Conductivity on Thermal Convection

2.1 Introduction

Thermal evolution of planetary mantles is influenced by many factors, such as temperature, viscosity (Tozer, 1972; Sharpe and Peltier, 1978) and nature of non-Newtonian rheology in the upper mantle (Christensen, 1985, Kawada and Honda, 1999). Influence of depth-dependent viscosity on structures of mantle convection have been studied by Gurnis and Davies (1986) and Hansen et al. (1993) in two-dimensional Cartesian boxes and by Zhang and Yuen (1995, 1996) and Bunge et al. (1996) in spherical-shell three-dimensional convection. Up to now, scant attention has been paid to potential influence of mantle thermal conductivity on the thermal evolution, although there have been numerous studies, which employed depth-dependent thermal conductivity (e.g., Leitch, et al., 1991; Balachandar, et al., 1992; Tackley, et al., 1996). These studies, however, contained other physical effects, such as variable viscosity and variable thermal expansivity, which prevented focussed and complete understanding of exact roles played by depth-dependent thermal conductivity.

Recently Hofmeister (1999, 2001) has developed a semi-empirical model, based principally on the phonon physics, to account for temperature- and pressure-dependent thermal conductivity of a whole suite of mantle materials. Hofmeis-
ter’s works have stimulated some recent modelling efforts of mantle convection in studying various facets where variable thermal conductivity can exert noticeable effects. These efforts include modelling of differences in three-dimensional planforms and interior temperatures (Dubuffet et al., 1999), differences caused by variable thermal conductivity on causing larger upwellings (Dubuffet and Yuen, 2000) and producing thinner downwellings (Dubuffet et al., 2000), influence on steady-state heat transfer (van den Berg et al., 2001), secular cooling of the mantle (van den Berg et al., 2002) and stabilizing influence on deep mantle plumes by high temperature at the core-mantle boundary (Dubuffet et al., 2002).

In these previous works, temperature-dependent thermal conductivity has produced lateral and vertical variations due to interactions with convection, which prevents understanding basic physics concerning the interactions with convection and variable thermal conductivity. DeLandro-Clarke et al. (1997) have, however, shown that secular cooling of a planet is more sensitive to vertical viscosity variations than to lateral variations, although temperature-dependent viscosity produces lateral and vertical variations in the interiors of planets. Their result has suggested that some features characterizing thermal convection, for example, cooling rate, averaged temperature and Nusselt number, are reasonably explained by vertical viscosity variations. On the other hand, van den Berg et al. (2002) have also shown that vertical thermal conductivity variations control some features characterizing thermal convection as vertical viscosity variations do.
In this work, I will take a different tack than previous investigations which employed the complete formula, given in Hofmeister (1999) in prescribing the explicit temperature- and pressure-dependence of thermal conductivity. In order to look at this problem from perspective of a purely depth-dependent stratification in the conductivity, I have devised a simplified two-layer mantle conductivity model. I will then carry out systematic studies by varying an interface separating two regions with vastly different mantle conductivity. Such a philosophy in varying the interface has its original roots in studies of mantle viscosity and rotational dynamics in which the interface separating two regions with different viscosity in the mantle was varied systematically with depth in the lower mantle (e.g. Yuen and Sabadini, 1985).

In section 2.3, I describe results for a two-dimensional model based on this two-layer conductivity model. I will in section 2.4 extend this two-layer conductivity concept to a complimentary approach in one-dimensional, using an one-dimensional loop model, which was developed first by Welander (1967) and employed later in mantle convection by Weinstein et al. (1989). Finally, in section 2.5, I will summarize the results and discuss ramifications for mantle convection.
2.2 Two-Dimensional Model

2.2.1 Basic Equations for Thermal Convection

For modeling mantle convection, I have considered a two-dimensional rectangular box filled with a very viscous fluid, and having an aspect ratio fixed at one (see Fig.2). Horizontal and vertical axes are denoted by \( x \) and \( z \), respectively. Horizontal boundaries are kept at a colder temperature \( T_s \) at the top and a hotter \( T_b \) at the bottom. The free-slip condition is also imposed at the horizontal boundaries. Lateral boundaries for the temperature and the complete velocity fields are reflective in character. I make the following assumptions, which are:

1. the mantle is an incompressible fluid, 
2. the Prandtl number is infinite, 
3. the Boussinesq approximation is valid.

Governing non-dimensional equations are derived from the conservation of mass, momentum and energy (Schubert et al., 2001):

\[
0 = -Ra_s \frac{\partial T}{\partial x} + \nabla^4 \psi, \tag{6}
\]

\[
\frac{\partial T}{\partial t} = -\left( \frac{\partial T}{\partial x} \frac{\partial \psi}{\partial z} - \frac{\partial T}{\partial z} \frac{\partial \psi}{\partial x} \right) + \nabla \cdot (k(z) \nabla T), \tag{7}
\]

where \( T(x, z, t) \) is the temperature field and \( \psi(x, z, t) \) is the stream function, which satisfies the conservation of mass automatically. \( k(z) \) is the non-dimensional depth-dependent thermal conductivity normalized by the surface value \( k_0 \). The second term in eq.(7) is the heat advection term. \( Ra_s \) is the Rayleigh number defined with the surface property values, expressed by eq.(9). In non-dimensionalizing
these equations, the following schemes based on a thermal diffusion time scale are used:
\[ x = Lx', \quad T = \Delta T', \quad t = \left( \frac{L^2}{\kappa_0} \right) t', \quad (8) \]
\[ Ra_s = \frac{\alpha g \Delta T L^3}{\kappa_0 \nu}, \quad (9) \]
where \( L \) is the thickness of the layer, \( \Delta T \) is the temperature difference imposed by the horizontal boundaries. \( \kappa_0 \) defined by \( k_0/\rho c \) is the thermal diffusivity estimated at the surface, \( \rho \) is the reference density, \( c \) is the specific heat, \( \alpha \) is the thermal expansivity and \( \nu \) is the kinematic viscosity.

2.2.2 Thermal Conductivity Model of the Mantle Rocks

A new model of thermal mantle conductivity, based on the phonon theory and infra-red spectroscopy, has recently been proposed by Hofmeister (1999, 2001). This model includes both lattice and radiative contributions, \( k_{\text{lat}} \) and \( k_{\text{rad}} \). This dimensional thermal conductivity model is based on experimental phonon lifetimes and infra-red reflectance data, which reads;
\[ k_{\text{lat}}(T, z) = k_{298} \left( \frac{298}{T} \right)^a \exp \left\{ -A_T + \frac{1}{3} \right\} \int_{298}^T \alpha(\theta) d\theta \left( 1 + \rho g' \frac{K' z}{K} \right), \quad (10) \]
\[ k_{\text{rad}}(T) = \sum_{i=0}^{3} b_i T^i, \quad (11) \]
where \( k_{298} \) is the value of thermal conductivity at \( T=298 \) K which is a value estimated for the Earth's surface, \( \gamma \) is the Grüneisen parameter, \( \alpha \) is the thermal expansivity, \( K' \) is the pressure derivative of the bulk modulus, \( K \) is the bulk
modulus, $b_i (i=0,1,2,3)$ are constants associated with diffusive radiative processes. The parameter $a$ in the lattice contribution eq.(10) is associated with types of chemical bondings in mantle minerals. The lattice contribution eq.(10) includes two nonlinear temperature dependences, that is, one is a decreasing exponential function and the other is a decreasing power-law dependence associated with a power-law index $a$, which depends on the mineralogy. As a result, $k_{\text{lat}}$ decreases rapidly, as $T$ increases with depth across the lithosphere. On the other hand, $k_{\text{rad}}$ increases rapidly with $T$ at the D’-layer. Hofmeister’s thermal conductivity model produces a very complicated distribution in the mantle and requires detailed analyses of the complex results.

First, I have introduced temperature-dependent thermal conductivity models to confirm that vertical thermal conductivity variations dominantly control some features characterizing thermal convection (see section 2.3.1). Thermal conductivities for this model are dependent on temperatures and variable both horizontally and vertically. Thermal conductivity models defined by horizontally averaged temperatures for above models are variable only vertically. Comparisons between results obtained based on these models indicate that vertical thermal conductivity variations play important roles on characteristic features such as averaged temperatures, averaged velocities and Nusselt numbers.

Next, in order to understand the multitudinous effects from various stratifications of thermal conductivity on mantle convection, I adopt a simpler model of depth-dependent thermal conductivity (see section 2.3.2 and section 2.3.3), in-
stead of taking on Hofmeister’s more complicated model (Dubuffet et al., 2000).

The basic equations are discretized by the central finite-difference method in the diffusion term, and by the Arakawa-Jacobian method (Arakawa, 1966) in the advection term. For boxes with aspect ratios fixed to one, numbers of vertical and horizontal grid points are 257 and 257, respectively. The energy equation is discretized by the explicit Euler method and integrated by the 4th-order Runge-Kutta method in time. The momentum equation is solved by the MGCG method (Tatebe, 1993).

2.3 Two Dimensional Results

2.3.1 Validity of Focus on Influence Caused by Vertical Variations of Thermal Conductivity

First, I will confirm that vertical thermal conductivity variations dominantly control some important features characterizing heat transfer of thermal convection. To do it, I compare three models whose thermal conductivity has different dependence on temperature (Fig.3, 4).

Fig.3 shows comparisons between models with constant thermal conductivity ($k=1$) and temperature-dependent lattice thermal conductivity. The lattice thermal conductivity decreases as temperature increases, which is expressed as $k(T) = (0.1/(0.1 + T))^{0.9}$, approximately similar to the form of the lattice thermal conductivity eq.(10) (Hofmeister, 1999, 2001). Comparison in structures of
temperatures and stream functions (Fig.3 (a), (b)) shows lower temperature and slower flow in the model with the lattice thermal conductivity. Comparison in vertical profiles of horizontally averaged temperature (Fig.3 (d), (e)) also shows similar tendency. The lattice thermal conductivity for figure (b) is dependent on temperature and varies both vertically and horizontally (Fig.3 (c)).

To consider roles of vertical and horizontal variations of thermal conductivity separately, I compare differences between two models whose thermal conductivity is dependnet on temperature and horizontally averaged temperature (Fig.4), respectively. Fig.4 (a) shows structure of temperature and stream function for a model whose lattice thermal conductivity is dependent on temperature; it varies both vertically and horizontally. Fig.4 (b) shows a model result for a model whose lattice thermal conductivity is dependent on horizontally averaged temperature; it varies only vertically. Comparison between Fig.4 (a) and (b) shows no significant differences for structures of temperatures and stream functions as a whole. Figs.4 (e) and (f) show vertical profiles of horizontally averaged temperature and thermal conductivity, respectively. These figures also show no significant differences between two models with different kinds of dependence on temperature.

Fig.4 (c) shows distribution of temperature difference which shows the temperature shown in Fig.4 (a) minus those for Fig.4 (b). In this figure, I found the model (a) has higher temperature in the ceneter of plumes, especially plume heads, and lower temperature around plume tails. These tendencies mean a more coherent hot plume and a more diffusive cold plume in the model with the hori-
zontal variation of the lattice thermal conductivity. However, I note a fact that amplitude of temperature variations is not very large. Fig.4 (d) shows distribution of thermal conductivity difference which shows the temperature of Fig.4 (b) minus those for Fig.4 (a). This figure denotes a horizontal variation of the lattice thermal conductivity. That is, the thermal conductivity is higher in a cold plume and lower in a hot plume. Such a horizontal variation of thermal conductivity can explain the tendencies shown in Fig. 4 (c): Lower thermal conductivity in an upwelling prevents the hot plume from being diffused. On the other hand, higher thermal conductivity in a downwelling promotes the cold plume to be diffused.

As I showed above, the influence of the temperature-dependent thermal conductivity can be separated into two kinds of influence associated with vertical and horizontal variations of the thermal conductivity. The main influence of variable thermal conductivity is a change of efficiency of heat transfer in thermal convection, which includes changes of averaged temperature and averaged stream function. This influence can be reproduced in a model without the horizontal variation of the thermal conductivity. That is, the vertical variation of the thermal conductivity controls efficiencies of heat transfer of thermal convection. On the other hand, the horizontal variation of the thermal conductivity affects efficiency of heat transfer insignificantly but shapes of plumes significantly. This may be important on problems about deformations caused by a convective flow.
2.3.2 Two-Layer Models of Thermal Conductivity

As I shown above, the vertical variations of the thermal conductivity dominantly control the efficiency of heat transfer of thermal convection. In this section, I develop a two-layer thermal conductivity model with simplified vertical variations. I focus on steady-state results with depth-dependent thermal conductivity.

Fig. 5 shows depth-dependent thermal conductivity models adopted here. There are two possibilities for thermal conductivity distribution. The first model has thermal conductivity $k=1$ in an upper layer and $k=0.5$ in an lower layer. Hereafter, I call this as the decreasing $k$ model. The second model has still $k=1$ in the upper part, while $k$ is increased to 1.5 in the lower layer. This model is called as the increasing $k$ model. I put the boundary separating two thermal conductivities ($k$-boundary) at various depths in order to investigate the dynamical effects of the depth-dependent thermal conductivity on mantle convection. Such an attempt has not been carried out before in mantle convection studies.

Fig. 6 shows relationships between depths of $k$-boundary and averaged temperature $T_{ave}$, averaged value of stream function $\psi_{ave}$ and the Nusselt number $Nu$ in the decreasing $k$ model. In cases where depths of the decreasing $k$-boundary are located at $z=0.05$ or 0.1, or close to the top (the lithosphere), there are both rapid decrease in $T_{ave}$ and rapid increase in $\psi_{ave}$ and $Nu$, as compared to a case with $k=0.5$ everywhere. On the other hand, in cases where depths for the decreasing $k$-boundary are located at $z=0.9$ or 0.95, or close to the bottom, there
are rapid decreasing changes in $T_{\text{ave}}$, $\psi_{\text{ave}}$ and $Nu$, as compared to a case with $k=1$ everywhere. These cases discussed above, especially, with $k$-boundary close to the top, may be valid for the Earth’s upper mantle, because thermal conductivity in the lithosphere is higher than in the upper mantle due to its lower temperature (Schatz and Simmons, 1972).

Likewise, Fig.7 shows relationships for models with increasing $k$-boundaries. In cases where depths of increasing $k$-boundary are located at $z=0.05$ or 0.1, there are rapid increase in $T_{\text{ave}}$ and rapid decrease in $\psi_{\text{ave}}$ and $Nu$, as compared to a case with $k=1.5$ everywhere. On the other hand, in cases where values of $k$ near the bottom are higher than those above, there are rapid increases in $T_{\text{ave}}$, $\psi_{\text{ave}}$ and $Nu$, as compared to a case with $k=1$ everywhere. These cases, especially, with $k$-boundary close to the bottom, may be valid for the Earth’s lower mantle, because thermal conductivity in the D”-layer can be higher due to presence of molten iron with high thermal conductivity. This enriched iron environment may compensate for a decrease in thermal conductivity in the D”-layer, as predicted by Hofmeister (1999). Molten iron can migrate into the lower mantle from the outer core, as proposed by Manga and Jeanloz (1996).

In presence of thin layers involving a significant contrast in the thermal conductivity, there are rapid changes in $T_{\text{ave}}$, $\psi_{\text{ave}}$ and $Nu$. As the Rayleigh number becomes higher, these thicknesses with sharp variations of averaged quantities are about 0.2 for $Ra_s=10^4$, 0.1 for $Ra_s=10^5$ and 0.05 for $Ra_s=10^6$. These layers characterize the horizontal thermal boundary layers. On the other hand, in cases
where $k$-boundaries are imposed not near the top and the bottom but in the interior, there are no significant differences. It is highly possible that values of thermal conductivity in horizontal thermal boundary layers have far more significant effects on thermal convection than the interior conductivity values. This is probably true because contribution of thermal conduction is strongest in the horizontal thermal boundary layers.

2.3.3 Three-Layer Models of Thermal Conductivity

As I have already showed in two-layer models of thermal conductivity, vertical variations of thermal conductivity can be reasonably separated into three-layer models, rather than two-layer models; upper and lower thermal boundary layers and the interior. I develop such a three-layer model, and then investigates structures of thermal convection with its model. Fig.8 shows the three-layer model of thermal conductivity for the Rayleigh number estimated at the surface $Ra_s$ equalling to $10^5$. Depths of boundaries between these three layers are based on the results with two-layer models. That is, thicknesses of the horizontal thermal boundary layers are 0.1 for $Ra_s = 10^5$. An area from $z=0$ to $z=0.1$ is called as the upper thermal boundary layer, whose thermal conductivity is $k_i$. An area from $z=0.1$ to $z=0.9$ is called as the interior, whose thermal conductivity is $k_c$. An area from $z=0.9$ to $z=1$ is called as the lower thermal boundary layer, whose thermal conductivity is $k_b$. In each three-layer model, any pair of thermal conductivity is fixed at one and the other is varied 0.2 from 1.8, which enables us to
understand influence of the thermal conductivity in each layer clearly.

Fig.9 shows relationships between value of thermal conductivity in each layer and averaged temperature $T_{\text{ave}}$, averaged stream function $\psi_{\text{ave}}$, and Nusselt number $Nu$. Averaged temperature $T_{\text{ave}}$ is independent of the thermal conductivity in the interior $k_c$. Averaged stream function $\psi_{\text{ave}}$ and Nusselt number $Nu$ decrease as $k_c$ increases. These features associated with $k_c$ can be explained by changes of effective Rayleigh number which is inversely proportional to the thermal conductivity. $T_{\text{ave}}$ decreases as the thermal conductivity in the upper thermal boundary layer $k_i$ increases, or as the thermal conductivity in the lower thermal boundary layer $k_b$ decreases. $\psi_{\text{ave}}$ and $Nu$ increase as $k_i$ or $k_b$ increases. Changes of $T_{\text{ave}}$, $\psi_{\text{ave}}$ and $Nu$ associated with $k_i$ and $k_b$ are larger than those for $k_c$.

The Vertical variations of the thermal conductivity also control horizontal scales of convective cells. To confirm it, I carried out numerical experiments using the three-layer models of the thermal conductivity with aspect ratios fixed at three. Fig.10 shows structures of convective cells with various $k_i$ and $k_c = k_b = 1$. The prediction for a model with higher thermal conductivity in the upper thermal boundary layer $k_i=1.8$ (Fig.10 (a)), has three convective cells whose aspect ratios are about one. A convective structure with constant thermal conductivity, $k_i = k_c = k_b = 1$ (Fig.10 (b)), has also three convective cells. Aspect ratios of these cells are almost one. In a model with lower thermal conductivity in the upper thermal boundary layer $k_i=0.2$ (Fig.10 (c)), a horizontal scale of convective cell is three. Fig.11 shows structures of convective cells with various $k_b$ and $k_c = k_i = 1$. 

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In a model with higher thermal conductivity in the lower thermal boundary layer $k_b=1.8$ (Fig.11 (a)), horizontal scales of convective cells are about one. A convective structure with constant thermal conductivity, $k_b = k_c = k_i = 1$ (Fig.11 (b)), has also three convective cells. Aspect ratios of these cells are also close to one. In a model with lower thermal conductivity in the lower thermal boundary layer $k_b=0.2$ (Fig.11 (c)), a horizontal scale of convective cell is three. Fig.12 shows structures of convective cells with various $k_c$ and $k_i = k_b = 1$. In a model with higher thermal conductivity in the interior $k_c=1.8$ (Fig.12 (a)), a horizontal scale of convective cell is three. This case is same as a situation whose thermal conductivity in both thermal boundary layers is lower than that in the interior. A convective structure with constant thermal conductivity, $k_c = k_i = k_b = 1$ (Fig.12 (b)), has three convective cells. Aspect ratios of these cells are almost one. A model with lower thermal conductivity in the interior, $k_b=0.2$ (Fig.12 (c)), has also three convective cells. Horizontal scales of these cells are also close to one.

As I found above, thermal conductivity models with $k_i, k_b < k_c$ produce convective structures with longer horizontal scales.

### 2.4 One-Dimensional Loop Model

In the two-dimensional numerical experiments, I found that values of the thermal conductivity in the horizontal thermal boundary layers exert far more significant
influence on heat transfer of mantle convection than values in the interior. I also
found that vertical variations of the thermal conductivity can vary horizontal
scales of convection cells, that is, lower thermal conductivity in thermal bound-
ary layers produces convective patterns with longer horizontal scales. These are
new findings, which have not been realized before in convection models using
Hofmeister’s conductivity model (e.g., Dubuffet et al., 1999, 2000; van den Berg
and Yuen, 2002). To understand the basic physics of these features caused by
thermal conductivity, I develop a loop model, which is computationally very effi-
cient and allows us to explore features of convection easily in the parameter space.
Loop models can reveal similar tendencies of thermal convection within a much
shorter computational time, by about a factor of 30, than two-dimensional mod-
els. This one-dimensional approach also enables us to obtain results with very
large aspect ratios much more easily. A loop model is a one-dimensional thermal
convection model, which is analogous to two-dimensional thermal convection by
making some simplifying physical assumptions and this simplified has allowed
much physical insight to be gleaned (Welander, 1967; Keller, 1966; Weinstein et
al., 1989; Guillou et al., 1995).

2.4.1 One-Dimensional Loop Model and its Assumptions

Following Weinstein et al. (1989), I have considered four pipes filled with fluid at
borders of a loop, shown in Fig.13. Vertical pipes, which are thermally insulated,
are analogous to an upwelling and a downwelling in two-dimensional convection.
Horizontal pipes, which are kept at constant temperature $T_{\text{bot}}$ and $T_{\text{top}}$ ($T_{\text{bot}} > T_{\text{top}}$), are analogous to the bottom and the top thermal boundary layers. A fluid in the interior of loop is stationary and isothermal at the mean temperature. I also introduce the downstream coordinate $s$, as in Welander (1967).

I have made some assumptions in constructing our loop model. The fluid is incompressible and the Boussinesq approximation is valid. The Prandtl number is infinite. The temperature and the velocity of the fluid are uniform over every cross-section of the pipes. The formulation of the loop model, as applied to mantle convection, was described in detail by Weinstein et al. (1989).

**2.4.2 Energy Equation of Loop Model**

In the loop model, heat flux is assumed to be proportional to temperature differences between the fluid and the pipes. In addition, I have neglected thermal interactions between the core region and the boundary layers (Weinstein et al., 1989). Then, the energy equation is written as,

$$\frac{dT}{dt} = D_{\text{top}}(T_{\text{top}} - T), \quad \text{at the top horizontal pipe,} \quad (12)$$

$$\frac{dT}{dt} = D_{\text{bot}}(T_{\text{bot}} - T), \quad \text{at the bottom horizontal pipe,} \quad (13)$$

$$\frac{dT}{dt} = 0, \quad \text{at vertical pipes,} \quad (14)$$

where $D_{\text{top}}$ and $D_{\text{bot}}$ are the coefficients of heat transfer with dimensions in sec$^{-1}$ at the top and the bottom horizontal pipes, respectively. $T$ is the temperature of
the fluid. $T_{\text{top}}, T_{\text{bot}}$ is the temperature of each horizontal pipe. These equations are shown diagrammatically in Fig.13.

Now I introduce the different heat transfer coefficients $D_{\text{top}}, D_{\text{bot}}$, which are analogous to the thermal conductivity, for the top and bottom horizontal pipes to compare the two-dimensional models with the one-dimensional loop models. I only consider the heat transfer between the fluid and the top and the bottom boundaries. Any other heat transfer mechanisms are neglected in the loop model. However, this limitation is not serious, because the two-dimensional results (see Figs.6, 7) showed that the values of thermal conductivity in the horizontal thermal boundary layers exert more significant effects. On the other hand, the interior values of the heat transport property are less influential on heat transfer of thermal convection.

2.4.3 Momentum Equation of Loop Model

Viscous force exerted on walls of the pipes by the fluid is proportional to the velocity (Weinstein et al., 1989). Then the momentum equation is expressed as,

$$0 = -\nabla p + \rho g - \rho_m R \nu,$$

(15)

where $R$ is the linear coefficient of viscous force with a dimension of sec$^{-1}$, $p$ is the total pressure, $g$ is the gravity acceleration and $\nu$ is the velocity of the fluid. $\rho$ is the density of the fluid and $\rho_m$ is the density of the fluid at the mean temperature $T_m$. The fluid has the Boussinesq relationship between the density
and the temperature,
\[ \rho = \rho_n [1 - \alpha (T - T_m)] \]  
(16)

### 2.4.4 Non-Dimensional Equations of the Loop Model

Since the fluid is assumed to be incompressible, the mass flow rate \( Q \) is a function only of \( t \). The temperature inside the loop \( T \) is a function of both \( t \) and \( s \). I have non-dimensionalized the equations (12), (13) and (14) with the following scaling;

\[ s = L s' \]  
(17)

\[ T = (T_{\text{bot}} - T_{\text{top}}) T' \]  
(18)

\[ Q = \int_A u dA = A L D_0 Q' \]  
(19)

\[ t = D_0^{-1} t' \]  
(20)

\[ D_{\text{top}} = D_0 D'_{\text{top}}, \quad D_{\text{bot}} = D_0 D'_{\text{bot}}, \]  
(21)

\[ \frac{M}{L} = \triangle \]  
(22)

where \( A \) is the cross-sectional area of the loop. \( D_0 \) is the reference value for the coefficient of heat transfer and primes mean non-dimensional equations. \( L \) and \( M \) are the lengths of the vertical and horizontal pipes respectively. \( \triangle \) is the horizontal scale of the loop (aspect ratio). I note that the time scale (20) cannot be used for comparing directly with the two-dimensional models, because of uncertainties in relationships between \( D \) in the loop models and the thermal conductivity \( k \) in the two-dimensional models. Omitting the primes for brevity,
I obtain the following non-dimensional equations.

\[
\frac{\partial T}{\partial t} = -Q \frac{\partial T}{\partial s} + D_{\text{bot}} (1 - T), \quad 0 \leq s \leq \Delta, \quad (23)
\]

\[
\frac{\partial T}{\partial t} = -Q \frac{\partial T}{\partial s} - D_{\text{top}} T, \quad \Delta + 1 \leq s \leq 2\Delta + 1, \quad (24)
\]

\[
\frac{\partial T}{\partial t} = -Q \frac{\partial T}{\partial s}, \quad \Delta \leq s \leq \Delta + 1, \quad 2\Delta + 1 \leq s \leq 2\Delta + 2, \quad (25)
\]

\[
Q = Rw (2 + 2\Delta)^{-1} \left( \int_{\Delta}^{\Delta+1} Tds - \int_{2\Delta+1}^{2\Delta+2} Tds \right), \quad (26)
\]

\[
Rw = \frac{g\alpha (T_{\text{bot}} - T_{\text{top}})}{D_0 RL}, \quad (27)
\]

where the non-dimensional parameter \(Rw\) is analogous to the Rayleigh number in the two-dimensional convection models.

### 2.4.5 Steady-State Solutions of the Loop Model

In the steady state by setting \(\frac{\partial}{\partial t} = 0\), I can solve the equations (23) to (26) analytically. I can write down the following equations in the steady-state limit:

\[
T_H = 1 - (1 - T_C) \exp(-D_{\text{bot}} \Delta / Q), \quad (28)
\]

\[
T_C = T_H \exp(-D_{\text{top}} \Delta / Q), \quad (29)
\]

\[
Q = \frac{Rw (T_{\text{bot}} - T_{\text{top}})}{2(1 + \Delta)}. \quad (30)
\]

In eq.(30), the flow rate \(Q\) is proportional to a ratio of buoyancy to friction. These simultaneous equations (28), (29) and (30) are solved numerically by an iteration method. \(D_{\text{top}}, \ D_{\text{bot}}, \ \Delta\) and \(Rw\) are given as input parameters. Temperatures of the upwelling \(T_H\) and the downwelling \(T_C\) and the flow rate \(Q\) are unknowns to be solved.
Fig. 14 shows basic results of a steady-state loop model. As a horizontal scale of the loop \( \Delta \) goes to zero, the flow rate \( Q \) also goes to zero and \( T_H \) and \( T_C \) approach an average temperature \( T = 0.5 \). Because the horizontal scale of the loop is small, total viscous friction exerted on the fluid is also small. In such cases with too short horizontal scales, the fluid can not be heated or cooled and therefore \( T_H \) and \( T_C \) for plumes in each horizontal pipe go to 0.5. That is, a temperature difference between \( T_H \) and \( T_C \) becomes small, resulting in less buoyancy of the fluid. In a limit with too short horizontal scale loop, the buoyancy is small and cannot drive a convective flow even though the total friction is also small. In another limit that a horizontal scale of the loop \( \Delta \) is lengthened infinitely, \( Q \) also goes to zero, and \( T_H \) goes to \( T_{\text{bot}} = 1 \), and \( T_C \) also goes to \( T_{\text{top}} = 0 \). Because the horizontal scale of the loop is large, the total friction exerted on the fluid is also large. In such cases with too long horizontal scales, the fluid can be heated or cooled enough while the fluid flows in each horizontal pipe. That is, \( T_H \) goes to one and \( T_C \) goes to zero, and then a temperature difference between \( T_H \) and \( T_C \) becomes maximum, resulting in maximum buoyancy. In a limit with too long horizontal scale loop, the total friction is large and a convective flow decays even though the buoyancy is maximum.

2.4.6 Influence Caused by Different Heat Transfer Coefficients

Fig. 15 shows results of models with decreasing heat transfer coefficients. In a case in which \( D_{\text{top}} \) is higher than \( D_{\text{bot}} \) (dotted line), temperature of plumes,
especially, \( T_H \) is lower than a reference value estimated in a model with \( D_{\text{top}} = D_{\text{bot}} \) (solid line). This implies that a decreasing \( D_{\text{bot}} \) causes a weaker heat transfer from the bottom. Consequently, the fluid in the lower pipe cannot be heated enough and has less buoyancy, causing a decrease in the flow rate. In a case that both \( D_{\text{top}} \) and \( D_{\text{bot}} \) are decreasing (dashed line), \( T_H \) decreases, whereas, \( T_C \) increases, as compared to the reference value. This implies that a decreasing \( D_{\text{top}} \) weakens heat transfer from the top. Consequently, the fluid in upper pipe can not be cooled enough, causing a slower flow rate. The weaker heat transfer in the horizontal pipes requires a longer horizontal scale of loop to obtain enough thermal buoyancy for achieving a maximum in the flow rate. This feature is consistent with the longer horizontal scales with a decreasing \( k \) found in the two-dimensional results, provided I assume that a convective structure with a maximum flow rate is also found in the two-dimensional results.

Fig. 16 shows results of loop models with increasing heat transfer coefficients, which is more like cases of the Earth’s mantle. In a case in which \( D_{\text{bot}} \) is higher than \( D_{\text{top}} \) (dotted line), the temperature of plumes, especially, \( T_H \) increases from the reference (solid line). This implies that an increasing \( D_{\text{bot}} \) causes a stronger heat flow from the bottom. The fluid in the lower pipe is heated excessively and derives more buoyancy, which causes an enhancement in the flow rate. In a case that both \( D_{\text{top}} \) and \( D_{\text{bot}} \) are increasing (dashed line), \( T_H \) increases. On the other hand, \( T_C \) decreases, compared with the reference value. This implies that an increasing \( D_{\text{top}} \) causes a stronger heat transfer at the top. Thus, the fluid in
upper pipe is cooled excessively, which, in turn, delivers a much faster flow rate. The stronger heat transfer in the horizontal pipes needs only a shorter horizontal scale of loop to gain enough thermal buoyancy for achieving a maximum in the flow rate. These dynamical features are also consistent with the two-dimensional results.

2.5 Discussion and Conclusions

I have studied the basic dynamical influence of the depth-dependent thermal conductivity on thermal convection by carrying out numerical experiments with both two-dimensional cartesian convection model and one-dimensional loop model. As a preparatory step, I have shown that the vertical thermal conductivity variations dominantly control some important features characterizing thermal convection focused in this paper. To simplify the physics, I have used a two-layer model of the thermal conductivity whose boundary is imposed at various depths. I have complemented the results obtained from two-dimensional results by developing a one-dimensional loop model to understand the physics under a simpler circumstance. I have found that the values of the thermal conductivity in the horizontal thermal boundaries can exert significant influence on convection. My findings are summarized in the order of their importance for mantle convection: (1) the particular values of the thermal conductivity in the upper thermal boundary layer, or the presence of a low conductivity zone under the lithosphere (van den Berg and
Yuen, 2002), which controls the strength of diffusion relative to advection there and influences the growth of the upper thermal boundary layer and the development of the cold downwelling (Dubuffet et al., 2000), (2) the particular values of the thermal conductivity in the lower thermal boundary layer (e.g., Brown, 1986), or the D’-layer (Leitch, 1995) in the Earth’s mantle, which controls the strengths of diffusion relative to advection there and influences the growth of the lower thermal boundary layer and the development of the hot upwelling in the lower mantle, (3) the specific values of the thermal conductivity in the lower and upper mantle outside the thermal boundary layers (Leitch, 1995), which control the diffusion of buoyancy of plumes and influence the vigor of convection by changing the effective Rayleigh number of the mantle.

Hofmeister’s new model of mantle thermal conductivity shows a more rapid decrease as temperature increases at relatively low temperatures than the other $k$ models (e.g., Schatz and Simmons, 1972). This physical characteristic suggests that the thermal conductivity decreases rapidly near the upper thermal boundary, that is, the lithosphere. In this case, the lithosphere may be able to grow easily and produce a larger and heavier slab driving mantle convection. This appears to be consistent with lithospheric motions induced by mantle dynamics and enhance the contributions of the sinking the lithosphere to mantle dynamics (e.g., Davies, 1986).
3 Influence of Temperature-Dependent Thermal Conductivity on Thermal Convection with Temperature-Dependent Viscosity

3.1 Introduction

Recently, Hofmeister (1999, 2001) has proposed a new semi-empirical model based on the solid-physics to account for temperature- and pressure-dependent thermal conductivity of mantle materials. Some studies of thermal convection with variable thermal conductivity based on Hofmeister’s model have been carried out in applications of mantle convection (Dubuffet et al., 1999; Dubuffet and Yuen, 2000; Dubuffet et al., 2000; van den Berg et al., 2001; van den Berg et al., 2002; Dubuffet et al., 2002). They found some interesting results, for example, the changes in thinknesses of plumes (Dubuffet and Yuen, 2000; Dubuffet et al., 2000), in the efficiency of heat transfer (van den Berg et al., 2001; van den Berg et al., 2002) and in the stability near the bottom (CMB) (Dubuffet et al., 2002). Yanagawa et al. (2004) also have studied effects of the depth-dependent thermal conductivity by using the simplified stratifications of the thermal conductivity and found that actual values of the thermal conductivity in the horizontal thermal boundary layers can exert the significant influence on volumetrically averaged temperature, velocity and heat flux of thermal convection than values outside these areas (see section 2). However, all of these models have not in-
cluded influence of variable viscosity, which evidently plays an important role in mantle convection. Therefore it may be difficult to apply their constant viscosity results to the mantle.

In this section, I have set out to study mantle convection with temperature-dependent variable viscosity and temperature-dependent thermal conductivity. The lattice contribution of thermal conductivity decreases rapidly with increasing temperature, especially at low temperatures as expected to the lithosphere, for example, $T < 1440$ K for depth above 70 km (Schatz and Simmons, 1972; Hofmeister, 1999). On the other hand, the radiative contribution of thermal conductivity increases rapidly with increasing temperature, especially at high temperatures, for example, $T > 1870$ K for depth below 670 km (Schatz and Simmons, 1972; Hofmeister, 1999; Clark, 1957). The temperature-dependent viscosity also decreases as temperature increases, and the various viscosity contrasts across the layer produce some distinct kinds of patterns in mantle convection (Solomatov, 1995; Kameyama and Ogawa, 2000). Therefore, I study the influence of combination of the temperature-dependent thermal conductivity and the temperature-dependent viscosity, and compare them with results obtained by Solomatov (1995) and Kameyama and Ogawa (2000).

In section 3.2, I introduce two-dimensional models with variable viscosity and thermal conductivity and then describe results with both temperature-dependent viscosity and thermal conductivity in section 3.3. Finally, I summarize results and apply them to the mantles in other planetary bodies in section 3.4.
3.2 Model description

I consider thermal convection of a Newtonian fluid with strongly temperature-dependent viscosity and temperature-dependent thermal conductivity in a two-dimensional rectangular box heated from below (Fig.2). A temperature difference between the horizontal boundaries is fixed at one. The free-slip condition is imposed at the horizontal boundaries, and the lateral boundaries are reflective. The following assumptions on the fluid are imposed, that is, (1) the fluid is incompressible, (2) the Prandtl number is infinite, (3) the Boussinesq approximation is valid, i.e., no mechanical heating is included. These conditions are same as those considered in section 2.

3.2.1 Basic Equations

Governing non-dimensional equations are derived from the conservation of mass, momentum and energy (Schubert et al., 2001):

\[
\left( \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2} \right) \nu(T) \left( \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2} \right) \psi + 4 \frac{\partial^2}{\partial x \partial y} \nu(T) \frac{\partial^2}{\partial x \partial y} \psi = -Ra_s \frac{\partial T}{\partial x}, \quad (31)
\]

\[
\frac{\partial T}{\partial t} = - \left( \frac{\partial T \partial \psi}{\partial x \partial z} - \frac{\partial T \partial \psi}{\partial z \partial x} \right) + \nabla \cdot (\kappa(T) \nabla T), \quad (32)
\]

where \( T(x, z, t) \) is the temperature field, \( \psi(x, z, t) \) is the stream function, which satisfies the conservation of mass automatically. \( Ra_s \) is the Rayleigh number based on surface property values defined at the top. The second term in eq.(32) is the advection term. \( \nu(T) \) and \( \kappa(T) \) are non-dimensional temperature-dependent kinematic viscosity and temperature-dependent thermal diffusivity normalized by
the surface values, respectively. Thermal diffusivity $\kappa$ is proportional to thermal conductivity $k$, $\kappa(T) = k(T)/\rho_0 c$, where $\rho_0$ is the reference density and $c$ is the specific heat.

### 3.2.2 Fomulation of Temperature-Dependent Viscosity

The viscosity, adopted in this study, $\mu(T) = \rho_0 \nu(T)$, decreases exponentially with increasing temperature;

$$\nu(T) = \mu(T) \frac{\rho_0}{\rho_0} = \frac{\nu_s}{\rho_0} \exp[-\beta(T - T_s)], \quad \text{(33)}$$

where $\nu_s$ and $T_s$ are the viscosity and temperature at the top boundary, and $\beta$ is a constant controlling the viscosity contrast across the layer. The exponential formula of the temperature-dependent viscosity expressed as eq.(33) is called the Frank-Kamenetskii approximation (Frank-Kamenetskii, 1969). From now on, the kinematic viscosity $\nu$ is called as the viscosity simply.

### 3.2.3 Fomulation of Temperature-Dependent Thermal Conductivity

The lattice thermal conductivity also decreases as temperature increases. The relationship between the thermal conductivity and temperature takes the following power-law which is derived by simplifying the thermal conductivity model proposed by Hofmeister(1999), eq.(10);

$$k(T) = k_s \left( \frac{T_s}{T_s + T} \right)^a, \quad \text{(34)}$$

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where $k_s$ is the thermal conductivity at the top boundary, and $a$ is a constant, which is associated with types of chemical bondings in mantle minerals (Hofmeister, 1999). Now, $T_s$ is fixed at 0.1.

I also adopt the radiative thermal conductivity model which increases as temperature increases. The radiative thermal conductivity is proportional to the temperature cubed $T^3$ basically (Clark, 1957), and the radiative contribution of Hofmeister model eq. (11) has a similar dependence on temperature.

$$k(T) = 1 + fT^3,$$  \hspace{0.5cm} (35)

where $f$ is a constant and fixed at 4, which controls strength of the dependence on temperature.

### 3.2.4 Non-Dimensional Parameters Characterizing the Models

I adopt two non-dimensional parameters to understand the physics in problems, i.e., the Rayleigh number estimated on the top $Ra_s$, a viscosity contrast $\gamma_\nu$ between the top and the bottom;

$$Ra_s = \frac{\rho_0 g \alpha (T_b - T_s) d^3}{\kappa_s \mu_0},$$  \hspace{0.5cm} (36)

$$\gamma_\nu = \exp[-\beta(T_b - T_s)],$$  \hspace{0.5cm} (37)

where index “$b$” means the value evaluated at the bottom. $g$ is the gravitational acceleration, $\alpha$ is the thermal expansivity and $d$ is a thickness of the box.
3.2.5 Numerical Methods

The basic equations are discretized by the central finite-difference method in the diffusion term and the Arakawa-Jacobian method in the advection term. For boxes with aspect ratios fixed at one, the numbers of vertical and horizontal grid points are 257 and 257, respectively. The energy equation is discretized by the Euler method in time. The momentum equations with the temperature-dependent viscosity are solved by an iteration method. The accuracy of results is verified by a benchmark comparison with the results of Christensen (1984). In all models, the initial condition is at rest and has an isotherm at \( T = 1 \) with a very small disturbance in the center of the box. All models have been integrated until the volumetrically averaged temperature becomes almost steady.

3.3 Results

3.3.1 Convective Regimes of Thermal Convection with Temperature-Dependent Viscosity and Constant Thermal Conductivity

The Previous Works Before considering structures of thermal convection affected by interactions between the temperature-dependent viscosity and the temperature-dependent thermal conductivity, I examine convective structures of thermal convection for models with the temperature-dependent viscosity and constant thermal conductivity. Those convective patterns have already been studied very well. The temperature-dependent viscosity promotes the upper cold ther-
mal boundary layer to be a stiffer-lid as the viscosity contrast across the layer increases. Solomatov (1995) carried out simple scaling analysis of thermal convection with the temperature-dependent viscosity and constant thermal conductivity and predicted three convective modes: the small viscosity contrast regime which has no stiff-lid (SVC), the transitional regime which has a less-stiff lid (TR) and the stagnant-lid regime which has a stiff-lid (ST). Moresi and Solomatov (1995) confirmed the prediction by Solomatov (1995) by carrying out two-dimensional numerical experiments whose aspect ratios are fixed at one. Kameyama and Ogawa (2000) carried out two-dimensional numerical experiments of thermal convection with longer horizontal scales than models of Moresi and Solomatov (1995), and found similar convective regimes. In their numerical experiments, the viscosity are fixed at the minimum on the bottom boundary where temperature is maximum, and the viscosity increases as temperature decreases. Then, the upper cold thermal boundary layer becomes thicker and a convective flow becomes slower as the viscosity contrast increases. On the other hand, the viscosity are fixed at the maximum on the top boundary where temperature is minimum, and the viscosity decreases as temperature increases in this paper. In this model, the upper thermal boundary layer becomes thinner and a convective flow becomes faster as the viscosity contrast increases. So, I consider some convective modes in the models which have different viscosity condition from the previous works.
Plots of $< u_s >$ and $\textit{Nu}$ versus $\gamma_\nu$. Fig.17 shows plots of averaged horizontal velocity at the surface $< u_s >$ and Nusselt number $\textit{Nu}$ versus the viscosity contrast across the layer $\gamma_\nu$. The numerical experiments plotted in this figure are shown on Table 1. $< u_s >$ is expressed in models with aspect ratios fixed at one as follows,

$$< u_s > = \int_0^1 u_s dx.$$  \hfill (38)

Plots of $< u_s >$ can be divided into four areas whose gradients of the plots are different from each other: The area-I where the viscosity contrast $\gamma_\nu$ is under $10^{2.6}$ and $< u_s >$ increases as $\gamma_\nu$ increases, the area-II whose $\gamma_\nu$ is from $10^{3.01}$ to $10^{3.9}$ and $< u_s >$ decreases as $\gamma_\nu$ increases, the area-III whose $\gamma_\nu$ is from $10^{4.34}$ to $10^{5.21}$ and $< u_s >$ keeps almost same value, independent of $\gamma_\nu$, and the area-IV whose $\gamma_\nu$ is over $10^{5.61}$ and $< u_s >$ decreases as $\gamma_\nu$ increases. In plots of $\textit{Nu}$, the boundaries of these four areas, especially, the boundary between the area-II and the area-III, are less distinct.

**Definition of Viscous Dissipation** To recognize differences between these four areas, I examine convective structures of them and pay attention to distribution of viscous dissipation $\Phi/\Phi_{\text{ave}}$. The viscous dissipation $\Phi/\Phi_{\text{ave}}$, which is normalized by averaged viscous dissipation, is expressed as follows,

$$\Phi = \tau_{ij} \frac{\partial u_i}{\partial x_j} = \frac{\nu(T)}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2,$$  \hfill (39)

$$\frac{\Phi}{\Phi_{\text{ave}}} = \frac{\Phi}{\nu \int V \Phi dV},$$  \hfill (40)

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where \( \tau_{ij} \) is the shear stress, and \( V \) is the volume of the layer. There are significant deformations caused by a convective flow in zones where large viscous dissipation is distributed.

**Comparison between Cases A-C14 and A-C06**  
Fig.18 shows comparison of convective structures between cases A-C14 and A-C06. Case A-C06 is in the area-I. In case A-C06, the maximum viscous dissipation distributes at the root of the downwelling, where the depth \( z \) equals zero. This tendency means that there is no stiff-lid near the surface. Such a convective pattern is similar to the one with constant viscosity, and is therefore called as the small viscosity contrast (SVC) regime (Solomatov, 1995). The thermal convection in the icy outer shell of Galilean satellites seems to belong to the SVC mode. Case A-C14 is in the area-IV. In case A-C14, the maximum viscous dissipation indicating the root of the cold plume distributes near the bottom of the upper thermal boundary layer. There is also no significant viscous dissipation along the surface. In addition, there is small disturbance near the bottom of the upper thermal boundary layer. Such tendencies mean that there is little deformation near the surface, that is, a stiff-lid appears along the top. In other words, the fluid is almost stationary near the surface. Such a convective regime is called as the stagnant-lid (ST) regime. Thermal convection which belongs to the ST-regime is valid for the mantle convection in Mars or Venus. In those planets, the plates are very thick and stiff, and so are not deformed by the underlying mantle flow. That is, there
are no plate-tectonics on those planets (Moresi and Solomatov, 1995; Solomatov and Moresi, 1996).

Fig. 19 shows vertical profiles of temperature \( T \), viscosity \( \nu \) and horizontal velocity \( u/u_{ave} \) at the center \( x=0.5 \) in order to compare model results belonging to the SVC (A-C06) and the ST (A-C14) regimes. The horizontal velocity \( u/u_{ave} \) is normalized by the averaged horizontal velocity, which is expressed as,

\[
u_{x=0.5}/u_{ave} = \frac{u_{x=0.5}}{\frac{1}{V} \int_{V} u \, dV}.
\]

The layer where \( T \) is uniform and independent of depth \( z \) is the convective layer underlying below the upper thermal boundary layer. In the upper thermal boundary layer belonging to the ST regime, a temperature gradient is steeper. In a case with the same Rayleigh numbers (estimated at the surface), the convective layer belonging to the ST regime has the larger effective Rayleigh number than that for the SVC regime. The larger effective Rayleigh number promotes the upper thermal boundary layer to become thinner, and then the temperature gradient becomes steeper in the layer. A viscosity gradient reflecting the temperature profile is also steeper there. Because the diffusive terms of momentum in eq.(31) include the terms proportional to the viscosity gradient, the convective flows are decayed very much in the upper thermal boundary layer. As a result, the upper thermal boundary layer belonging to the ST regime becomes isolated dynamically from the underlying convective layer, corresponding to the stagnant-lid.
The small disturbances characterizing the ST regime are caused by the increased effective Rayleigh number due to the large temperature gradient and low viscosity around the bottom of the upper thermal boundary layer.

Comparison between Cases A-C12 and A-C08  Fig.20 shows comparison of convective patterns between cases A-C12 and A-C08. Cases A-C12 and A-C08 are in the area-III and area-II, respectively. In both patterns, the maximum viscous dissipation exists not at the surface but at the roots of the downwellings, and the significant viscous dissipation distributes vertically to the surface (A-C12), or horizontally in the lowermost part of the upper thermal boundary layer (A-C08). There is no small disturbance around the bottom of the upper thermal boundary layer. These tendencies mean that there is a less stiff-lid along the surface but the lid is not isolated dynamically from the underlying convective flow. In other words, the less stiff-lid can be deformed by the underlying convective flow. Such convective patterns are called as the transitional (TR) regime. There are plate-tectonics on the Earth, that is, the lithospheres of the Earth can be deformed by pulls of downwellings and move by the underlying mantle flow. So, the mantle convection on the Earth seems to belong to the TR regime (Solomatov, 1995).

The TR regime can be separated into two convective regimes. I can find the difference between the two regimes by comparing the model results for the area-II (A-C08) and the area-III (A-C12). For the area-II (A-C08), the viscous
dissipation caused by the downwelling distributes vertically to the surface. This means that the upper thermal boundary layer can be deformed mainly by the pull of the downwelling. I call such a mode as the TR-I regime. For the area-III (A-C12), the viscous dissipation distributes horizontally in the lowermost part of the upper thermal boundary layer. This means that the upper thermal boundary layer is deformed mainly by the underlying horizontal flow, not by the pull of downwelling. In other words, the pull of the downwelling cannot affect the deformation near the surface. I call this modes as the TR-II regime. The differences between the TR-I and the TR-II regimes can be also characterized by the different viscosity gradients in the upper thermal boundary layers. Because the TR-II regime has the larger viscosity contrast, the effective Rayleigh number becomes larger. As a result, the thermal boundary layer becomes thinner, and the viscosity gradient becomes steeper in the layer for the TR-II regime. Such a steeper viscosity gradient decays the flow in the upper boundary layer (Fig.21). In the Earth, the Pacific plate may be in the similar situation with the TR-I regime, and the spreading sea floors in the Atlantic Ocean may be in the same situation with the TR-II regime. The sea floors in the Atlantic Ocean are younger and thinner plates. In such plates, the viscosity gradient is steeper. So, those plates are driven by the underlying horizontal mantle flow, but cannot be subducted into the mantle because the downwelling cannot pull them downward. On the other hand, the Pacific plate is older and thicker. In this plate, the viscosity gradient is more gentle, and then the plate can be subducted into the mantle by
the pull of descending plume.

**Summary** As showed in this section, the patterns of thermal convection with the temperature-dependent viscosity can be separated into the four regimes: SVC, TR-I, TR-II and ST regimes. These four regimes can be found easily on the plots of averaged horizontal velocity at the surface versus the viscosity contrast across the layer. The four regimes are also characterized by the viscosity gradients in the upper thermal boundary layer. There is no lid along the surface in the convective structure for models with less viscosity contrast (SVC). As the viscosity contrast $\gamma_{\nu}$ increases, the upper thermal boundary layer becomes less mobile (TR-I). As the $\gamma_{\nu}$ increases further more, the effective Rayleigh number increases and then the upper thermal boundary layer becomes thinner. As the thermal boundary layer becomes thinner, the gradients of temperature and the viscosity become steeper in the layer and the diffusion terms of momentum become stronger. As a result, the pull of the downwelling cannot affect on the deformation close to the surface (TR-II). Finally, the upper thermal boundary layer becomes dynamically isolated from the underlying convective layer. This is a thin and stagnant-lid along the surface (ST). In this model, the change of $<u_z>$ with increasing $\gamma_{\nu}$ is decided by the balance between two contributions: One is due to the increase of the effective Rayleigh number, and the other is due to the increase of the viscosity gradients in the upper thermal boundary layer. The former increases $<u_z>$, and the latter decreases $<u_z>$ as $\gamma_{\nu}$ increases.
On the other hand, the separation into three convective regimes is adopted in the previous works (Solomatov, 1995; Kameyama and Ogawa, 2000). In their models with fixed minimum viscosity at the bottom, the effective Rayleigh number decreases and the viscosity near the surface increases as the viscosity contrast $\gamma_\nu$ increases. The low effective Rayleigh number weakens a convective flow, and the upper thermal boundary layer becomes thinner because of the larger viscosity close to the surface. Because such a thick thermal boundary layer cannot be deformed by a weakened convective flow, a stiff-lid appears along the surface. In their model, $< u_\delta >$ becomes slower monotonously as $\gamma_\nu$ increases.

Because the boundary condition of the viscosity is different between the previous and present works, the mechanisms that the convective regimes change from the SVC regime to the ST regime as the viscosity contrast increases, are also different. The transition from TR-I to TR-II regimes with increasing viscosity contrast is a new result shown in this paper.

### 3.3.2 Influence of the Lattice Thermal Conductivity on Thermal Convection with Shorter Horizontal Scale

In this section, I consider the influence of the temperature-dependent lattice thermal conductivity on thermal convection with the temperature-dependent viscosity, especially, on the deformation in the upper thermal boundary layer. The lattice thermal conductivity decreases as temperature increases, as expressed in eq.(34).
Plots of $< u_s >$ and $Nu$ versus $\gamma/\nu$ with $Ra_s=10^{2}$  Fig.22 shows plots of horizontal velocity at the top $< u_s >$ and Nusselt number $Nu$ versus the viscosity contrast across the layer $\gamma/\nu$ to compare the results for constant and temperature-dependent lattice thermal conductivity models. The numerical experiments plotted in this figure are shown on Table 1. The plots for the lattice thermal conductivity have lower $< u_s >$ and lower $Nu$, which can be explained as shown in the section 2. However, there is an exception in the plots of $< u_s >$: $< u_s$ for the constant thermal conductivity with $\gamma/\nu = 10^{2.17}$ is lower than that for the lattice thermal conductivity. In a model where $\gamma/\nu$ is close to zero, thermal convection with constant thermal conductivity has small effective Rayleigh number, and so the fluid is almost at rest. On the other hand, the lattice thermal conductivity increases the effective Rayleigh number, and so the fluid can flow even if $\gamma/\nu$ is close to zero. In the plots of $< u_s >$, the most noticeable feature is the absence of the flat zone, which corresponds to the TR-II regime for the models with constant thermal conductivity. To understand why the TR-II regime disappears in the models with the lattice thermal conductivity, I examine the convective structures for two models with constant thermal conductivity and with the temperature-dependent lattice thermal conductivity.

Comparison between Cases A-L08 and A-C08  Under the condition that $\gamma/\nu$ equals to $10^{3.47}$, the convective structures belong to the TR-I mode in both models (Fig.23). The significant features are predicted in case A-L08, i.e., the
viscous dissipation distributes more vertically from the cold plume to the surface, which means that the contribution of the pull caused by the downwelling is larger for case A-C08. I also found that the intervals of the isotherm become wider close to the surface and narrower to the bottom of the upper thermal boundary layer in case A-L08. Fig.24 shows vertical profiles of temperature $T$, thermal conductivity $k$, viscosity $\nu$ and horizontal velocity $u/u_{ave}$ for the models shown in Fig.23. The layers whose $T$ is almost uniform and independent of depths are the convective layers underlying below the upper thermal boundary layers. In case A-L08, the upper thermal boundary layer is thinner than that for A-C08. This is caused by the increasing effective Rayleigh number due to the decreasing lattice thermal conductivity. The temperature gradient is linear in the thermal boundary layer for case A-C08. On the other hand, for case A-L08, the temperature gradients is more gentle close to the surface and steeper close to the bottom of the upper thermal boundary layer. This tendency reflects the profiles of thermal conductivity because vertical heat flow must be kept constant in the thermal boundary layer. The viscosity gradient also reflects the temperature gradient. In case A-L08, the viscosity gradient is more gentle close to the surface, where the convective flow is not decayed. This tendency is consistent with the profile of $u/u_{ave}$, where the gradient of $u/u_{ave}$ is more gentle close to the surface. On the other hand, the viscosity gradient is steeper close to the bottom of the upper thermal boundary layer, which denotes that the thermal boundary layer is more isolated dynamically from the underlying the convective layer.
Comparison between Cases A-L11 and A-C11  Fig.25 shows the two-dimensional convective structures with the viscosity contrast $\gamma_\nu$ fixed at $10^{4.77}$. In case A-C11, the viscous dissipation distributes horizontally in the lower part of the upper thermal boundary layer without small disturbance. Case A-C11 is in the TR-II regime. On the other hand, the viscous dissipation distributes vertically from the downwelling to the surface in case A-L11. So, case A-L11 has a feature characterizing the TR-I mode. In addition, the small perturbation appears around the bottom of the upper thermal boundary layer in case A-L11. This is the feature characterizing the ST regime. That is, the TR-I regime for the model with constant thermal conductivity is shifted into the other regime which has not only the features characterizing the TR-I but also the features characterizing the ST regimes. In this figure, I also found the same tendencies for the intervals of the isotherm with those found in Fig.23. In Fig.26, the vertical profiles of $T$, $k$, $\nu$ and $u/u_{ave}$ show the same tendencies with those found in Fig.24. The viscosity gradient is more gentle close to the surface, and steeper close to the bottom of the upper thermal boundary layer in case A-L11. The distribution of viscous dissipation (Fig.25) reflects the viscosity gradient in the upper thermal boundary layer for case A-L11. In case A-L11, $u/u_{ave}$ changes more rapidly in the narrower area, which indicates the existence of more isolated lid.

Comparison between Cases A-L15 and A-C15  Under the condition of $\gamma_\nu$ fixed at $10^{6.51}$, both conductivity models belong to the ST regime (Fig.27).
The viscous dissipation distributes only in the lowermost part of upper thermal boundary layer. There are also small perturbations around the bottom of the upper thermal boundary layer. In particular, in case A-L15, the downwellings become smaller and the number of cold plumes increases. The narrower intervals of the isotherm are also found around the bottom of the upper thermal boundary layer. This promotes the uppermost part of the upper thermal boundary layer to be stagnant. Fig.28 shows the vertical profiles of $T$, $k$, $\nu$ and $u/u_{ave}$ to compare the model results for lattice thermal conductivity (A-L15) and constant thermal conductivity (A-C15). The same tendencies with those found in Fig.26 are found in case A-L15, i.e., the thinner thermal boundary layer, the more gentle temperature gradient close to the surface and steeper temperature gradient around the bottom of the thermal boundary layer. As a result, viscosity gradient is also more gentle close to the surface and steeper around the bottom of the thermal boundary layer. Too steep viscosity gradient produces a dynamically isolated stagnant-lid along the surface in case A-L15. In other words, the pull of the downwelling cannot affect the deformation near the surface because the descending flows are decayed by the too steeper viscosity gradient in the lower part of the thermal boundary layer. The narrower layer where $u/u_{ave}$ changes rapidly indicates a more distinct boundary between the convective layer and the isolated stagnant-lid.
Plots of $<u_s>$ and $Nu$ versus $\gamma_\nu$ with $Ra_s = 10^3$. In Fig.29, the plots of averaged horizontal velocity $<u_s>$ and Nusselt number $Nu$ versus the viscosity contrast $\gamma_\nu$ are plotted in the cases with the larger Rayleigh number estimated at the surface $Ra_s$ fixed at $10^3$. The numerical experiments plotted in this figure are shown on Table 2. The similar features as found in the models with $Ra_s$ fixed at $10^2$ are also found. In the models with constant thermal conductivity, the area belonging to the TR-II is not distinct. The higher $Ra_s$ causes the powerful pull of downwellings, which extends the area belonging to the TR-I mode to the higher viscosity contrast zone. The higher $Ra_s$ also produces a thinner thermal boundary layer where the viscosity gradient is steeper, which extends the area belonging to the ST regime to the lower viscosity contrast zone. As a result, the area belonging to the TR-II regime is translated into the regime including features of both the TR-I and the ST regimes.

3.3.3 Influence of the Lattice Thermal Conductivity on Thermal Convection with Longer Horizontal Scale

Plots of $<u_s>$ and $Nu$ versus $\gamma_\nu$. Thermal convection with a proper value of $\gamma_\nu$ has a cold and thick thermal boundary layer along the surface, which prevents a fluid from being cooled. Therefore, a horizontal scale of such convection become longer than one because the fluid needs longer horizontal scale to be cooled enough to descend downward. On the other hand, thermal convection with relative small $\gamma_\nu$ has a less-developed upper thermal boundary layer, which doesn’t prevent the
fluid from being cooled. Thermal convection with relative large $\gamma_\nu$ has a large effective Rayleigh number, which increases efficiency of heat transfer. Therefore, a horizontal scale of such convection doesn’t become longer, because the fluid can be cooled enough through a upper thermal boundary layer while the fluid flows along the relative short surface. Aspect ratios of convective cells in the Earth’s mantle are probably larger than one, and so studies of thermal convection with longer horizontal scales than one are also useful to understand the dynamics of the Earth.

To consider structures of thermal convection with longer horizontal scales than one, aspect ratios of the boxes are fixed at two. For the thermal convection with longer horizontal scale fixed at two, the plots of the horizontal velocity at the surface $<u_s>$ and Nusselt number $Nu$ versus the viscosity contrast $\gamma_\nu$ are shown in Fig.30. The numerical experiments plotted in this figure are shown on Table 3. In the plots of $<u_s>$ with constant thermal conductivity, the three regimes, that is, the SVC, the TR and the ST regimes are found. Now, it is impossible to separate the TR regime into the TR-I and the TR-II regimes. In the thermal convection with longer horizontal scale, the cold plume becomes stronger because the fluid is sufficiently cooled while the fluid flows along the longer surface. This extends the area belonging to the TR-I regime to the larger viscosity contrast, and then the area belonging to the pure TR-II regime vanishes. On the other hand, there are no remarkable corners on the plot of the $<u_s>$ which denotes the boundaries between the convective regimes in the models with lattice thermal
conductivity.

**Comparison between Cases C-L10 and C-C10**  
Fig.31 shows the convective structures with $\gamma_\nu$ fixed at $10^{134}$. In case C-C10, the viscous dissipation distributes both horizontally along the bottom of the upper thermal boundary layer and vertically from the downwelling to the surface. That is, the convective structure belongs to the regime including features of both the TR-I and the TR-II regimes. For the convection with longer horizontal scale, the colder and heavier cold plume pulls the fluid near the surface, which adds the viscous distribution characterizing the TR-I regimes to the convective regime belonging to the TR-II regime. In case C-L10, the maximum viscous dissipation distributes at the foot of the downwelling and the significant viscous dissipation distributes vertically to the surface. This is the tendency characterizing the TR-I regime. In addition, there are small disturbance around the lowerpart of the upper thermal boundary layer, which is the feature belonging to the ST regime. That is, such a convective structure with the lattice thermal conductivity simultaneously has features characterizing the TR-I and the ST regimes. Fig.32 shows the vertical profiles of $T$, $k$, $\nu$ and $u/u_{ave}$ of the models shown in Fig.31. In case C-L10, the temperature profile shows that the upper thermal boundary layer is thinner and that the temperature gradient is more gentle close to the surface and steeper close to the bottom of the upper thermal boundary layer. The viscosity gradient has the same tendencies with the temperature gradient. The plot of $u/u_{ave}$ shows
the thicker layer where $u/u_{ave}$ is uniform near the surface, and the thinner layer where $u/u_{ave}$ is decayed rapidly near the bottom of the upper thermal boundary layer.

**Comparison between Cases C-L12 and C-C12**  Fig.33 shows the convective structures with $\gamma_\nu$ fixed at $10^5.21$. In case C-C12, the maximum viscous dissipation distributes at the root of the downwelling. In addition, the small perturbation appers at the bottom of the upper thermal boundary layer. These tendencies belong to the ST mode. The structure shown in case C-C12 is called as the elongated-ST regime by Kameyama and Ogawa (2000). The elongated-ST regime is the ST regime with longer horizontal scale. In case C-L12, the convective structure has the same tendencies with those found in Fig.31. That is, this convection has features characterizing the TR-I and the ST regimes, simultaneously. The vertical profiles of $T$, $k$, $\nu$ and $u/u_{ave}$ of the models shown in Fig.33 are depicted in Fig.34. These plots also show similar features with those shown in Fig.32.

**Comparison between Cases C-L14 and C-C14**  Fig.35 shows the convective structures with $\gamma_\nu$ fixed at $10^6.08$. Both cases C-L14 and C-C14 belong to the ST regime, because there is negligible viscous dissipation near the surface. In these cases, the aspect ratio of convective cells become shorter than two. There are small scale disturbances around the lowermost part of the upper thermal
boundary layer with the lattice thermal conductivity. The decreasing lattice thermal conductivity increases the effective Rayleigh number, and then convective pattern becomes more turbulent. This tendency is also shown in fig.27. Fig.36 shows the vertical profiles of $T$, $k$, $\nu$ and $u/u_{ave}$ of the models shown in Fig.35. These figures show the thinner and dynamically more isolated stagnant-lid with the lattice thermal conductivity as shown above.

**Summary** In section 3.3.2 and 3.3.3, the influence of temperature-dependent lattice thermal conductivity on thermal convection with temperature-dependent viscosity was investigated. The upper boundary layer becomes thinner because decreasing lattice thermal conductivity increases the effective Rayleigh number, and then the convective flow becomes stronger. In the boundary layer, the temperature and viscosity gradients become more gentle close to the surface, because the thermal conductivity is also higher close to the top. These tendencies enable the downwelling to pull the fluid near the surface. On the other hand, the temperature and viscosity gradients become steeper around the bottom of the upper thermal boundary layer, because the thermal conductivity is lower around there. These tendencies promote the upper thermal boundary layer to be isolated dynamically from the underlying convective layer. The characterizing convective structure with lattice thermal conductivity has features characterizing the TR-I and the ST regimes, simultaneously.
3.3.4 Influence of the Radiative Thermal Conductivity on Thermal Convection with Shorter Horizontal Scale

In this section, I investigate the influence of temperature-dependent radiative thermal conductivity on thermal convection with temperature-dependent viscosity. I especially focus the dynamics of the upper thermal boundary layer.

Plots of \(< u_s >\) and \( Nu \) versus \( \gamma_{\nu} \) with \( Ra_s = 10^2 \) Fig.37 shows the plots of averaged horizontal velocity at the surface \(< u_s >\) and Nusselt number \( Nu \) versus the viscosity contrast \( \gamma_{\nu} \) across the layer. The numerical experiments plotted in this figure are shown on Table 1. The Rayleigh number estimated at the surface \( Ra_s \) equals to \( 10^2 \), and the horizontal scales of the convective cells are fixed at one. The thermal convections with the radiative thermal conductivity produce faster \(< u_s >\) and higher \( Nu \) than those with constant thermal conductivity. In the plots of \(< u_s >\) for the models with the radiative thermal conductivity, I can also find four areas with different slopes of the plots as found in the plots with constant thermal conductivity. However, the difference between the area-II and the area-III is less distinct in the models with radiative thermal conductivity than those with constant thermal conductivity. In addition, the slope of the area-IV with the radiative thermal conductivity is steeper than that with constant thermal conductivity.
Comparison between Cases A-R08 and A-C08  

Fig. 38 shows the convective structures with $\gamma_\nu$ fixed at $10^{3.47}$. In case A-C08, the maximum viscous dissipation distributes at the foot of the cold plume, and the significant viscous dissipation distributes vertically to the surface. These tendencies belong to the TR-I regime. In case A-R08, the viscous dissipation distributes more horizontally along the bottom of the upper thermal boundary layer, more than vertically to the surface. This structure belong to the TR-II regime rather than the TR-I regime. The interval of the isotherms is narrower close to the top in case A-R08. Fig. 39 shows the vertical profiles of temperature $T$, thermal conductivity $k$, viscosity $\nu$ and horizontal velocity $u/u_{ave}$ of the models shown in Fig. 38. In case A-R08, the temperature profile shows hotter and thinner thermal boundary layer. However, the boundary between the thermal boundary layer and the convective layer is less distinct. As I demonstrated in section 2, the larger thermal conductivity close to the bottom of the box increases temperature of the fluid and strengthens the convective flow. The heated fluid has smaller viscosity, which increases the effective Rayleigh number. This strengthens the convective flow. 

As a result, the upper thermal boundary layer becomes thinner. In such a boundary layer, temperature gradient is steeper close to the surface, and more gentle around the lower part of the thermal boundary layer. Such a profile of temperature reflects the distribution of thermal conductivity. Because viscosity gradient reflects the temperature gradient, the viscosity gradient is steeper close to the surface, and more gentle around the bottom of the upper thermal boundary layer. The convective flow
is decayed close to the surface because the diffusive terms of the momentum in eq.(31) include the terms proportional to the viscosity gradient. The more gentle viscosity gradient around the lower part of the thermal boundary layer produces less distinct boundary between the thermal boundary layer and the convective layer.

**Comparison between Cases A-R12 and A-C12**  Fig.40 shows the convective structures with $\gamma_\nu$ fixed at $10^{5.21}$. Under the condition of $\gamma_\nu$, both cases A-R12 and A-C12 belong to the TR-II regime. The viscous dissipation distributes along the bottoms of the upper thermal boundary layers. In case A-R12, the narrower interval of the isothersms is consistent with the absence of significant viscous dissipation near the surface. In Fig.41, the vertical profiles of $T$, $k$, $\nu$ and $u/u_{ave}$ are shown. As I demonstrated in Fig.39, these profiles support the mechanism that the convective flow is decayed more close to the surface in case A-R12. On the other hand, the more gentle viscosity gradient produces less distinct boundary between the upper thermal boundary layer and the underlain convective layer.

**Comparison between Cases A-R14 and A-C14**  Fig.42 shows the convective structures with $\gamma_\nu$ fixed at $10^{6.51}$. Under the condition of $\gamma_\nu$, both cases A-R14 and A-C14 belong to the ST regime. The small disturbances apper around the bottoms of the upper thermal boundary layers. In Fig.43, the vertical profiles of $T$, $k$, $\nu$, and $u/u_{ave}$ are shown. As shown in Fig.41, for case A-R14, these
profiles support that the uppermost part in the upper thermal boundary layer is stiffer close to the surface, and the boundary between the lid and the underlying convective layer is less distinct.

**Plots of** $<u_s>$ **and** $Nu$ **versus** $\gamma_\nu$ **with** $Ra_s=10^3$  For the models with the larger Rayleigh number estimated at the surface $Ra_s$ fixed at $10^3$, the plots of averaged horizontal velocity $<u_s>$ and Nusselt number $Nu$ versus the viscosity contrast $\gamma_\nu$ are shown in Fig.44. The numerical experiments plotted in this figure are shown on Table 2. The features found in these plots are almost the same as those found in Fig.37. The difference between the TR-I and TR-II regimes is almost vanished in the plots with the radiative thermal conductivity. The slop of the ST regime with the radiative thermal conductivity is steeper. These features indicate that the radiative thermal conductivity promotes the uppermost part of upper thermal boundary layer to be stiffer.

### 3.3.5 Influence of the Radiative Thermal Conductivity on Thermal Convection with Longer Horizontal Scale

**Plots of** $<u_s>$ **and** $Nu$ **versus** $\gamma_\nu$  Fig.45 shows the plots of the averaged horizontal velocity $<u_s>$ and Nusselt number $Nu$ versus the viscosity contrast $\gamma_\nu$ for the models with longer horizontal scales fixed at two. The numerical experiments plotted in this figure are shown on Table 3. In the zone where $\gamma_\nu$ is below $10^{3.04}$, the values of the models with the radiative thermal conductivity
are not plotted because aspect ratios of their convective structures equals to not
two but one. The tendencies found in these plots are almost the same as those
found in Fig.37. That is, I found faster $<u_s>$, higher $Nu$ and steeper slope of
the ST regime in the plot with the radiative thermal conductivity.

Comparison between Cases C-R10 and C-C10  Fig.46 shows the convective structures with $\gamma_\nu$ fixed at $10^{4.34}$. In both cases C-R10 and C-C10, the maximums of the viscous dissipation distributes at the roots of the cold plumes around the bottoms of the upper thermal boundary layers. In case C-R10, the significant viscous dissipation distributes horizontally along the surface rather than vertically to the top. On the other hand, the vertical distribution of the viscous dissipation is more remarkable in case C-C10. There is less deformation near the surface in case C-R10, as we found in the models with the aspect ratios fixed at one. The vertical profiles of $T$, $k$, $\nu$, and $u/u_{ave}$ also support the upper thermal boudary layer which is stiffer close to the surface (Fig.47). That is, we can find the steeper viscosity gradient close to the surface, and more decayed $u/u_{ave}$ close to the surface in case C-R10.

Comparison between Cases C-R11 and C-C11  Fig.48 shows the two-dimensional convective structures and Fig.49 shows the one-dimesional convective structures for the condition with $\gamma_\nu$ fixed at $10^{4.77}$. In case C-C11, the distribution of the viscous dissipation shows the features characterizing the TR-I and the
ST regimes. The former is the vertical distribution from the cold plume to the surface, and the latter is the small perturbation around the bottom of the upper thermal boundary layer. This mode belongs to the elongated-ST regime characterized by the stagnant-lid, more grewed cold plume and longer horizontal scale (Kameyama and Ogawa, 2000). On the other hand, case C-R11 also belongs to the elongated-ST regime, but stiffer along the surface. The features found in the two-dimensional structures are supported by those found in the one-dimensional profiles, as demonstrated above.

Comparison between Cases C-R14 and C-C14  Fig.50 and Fig.51 show the two-dimensional convective structures and vertical profiles of some properties with $\gamma_0$ fixed at $10^{0.08}$, respectively. Both cases C-R14 and C-C14 belong to the ST regimes, which cannot keep their aspect ratio of the convective cells longer, because of the less contribution of the pull caused by the downwelling. I also found that the small perturbation is more diffused due to the higher value of the radiative thermal conductivity for case C-R14.

Summary  In section 3.3.4 and 3.3.5, the influence of the radiative thermal conductivity on the thermal convection with temperature-dependent viscosity, the deformation of the upper thermal boundary layer, was especially discussed as demonstrated above. The upper boundary layer becomes thinner because increasing radiative thermal conductivity increases the temperature of the fluid
and strengthens the convective flow. The more heated fluid has lower viscosity, which also strengthens the flow. In the boundary layer, the temperature and viscosity gradients become more steeper close to the surface, because the thermal conductivity is also lower close to the top. Because of these tendencies, the pull of the downwelling cannot contribute the surface deformation. On the other hand, the temperature and viscosity gradients become more gentle around the bottom of the upper thermal boundary layer, because the thermal conductivity is higher around there. These tendencies promote the lower part of the upper thermal boundary layer to be included into the underlying convective layer. The convective structure with radiative thermal conductivity is characterizing a stiffer uppermost part of the lid.

3.4 Discussion and Conclusions

In this section, I have studied the combined effects of the temperature-dependent variable thermal conductivity and the temperature-dependent viscosity on mantle convection by carrying out two-dimensional numerical experiments. I focus on the dynamics of the lithosphere affected by the temperature-dependent thermal conductivity.

3.4.1 Influence of the Lattice Thermal Conductivity

I first investigated the influence of the temperature-dependent lattice thermal conductivity on thermal convection with the temperature-dependent viscosity.
The thermal convection with the lattice thermal conductivity has lower temperature and slower convective flow, which is explained as discussed in section 2. The contribution of the lattice thermal conductivity is emphasized in the cases with larger viscosity contrasts. This means that there is significant influence in a stiff lid near the surface appeared in models associated with the TR-I, TR-II and ST regimes. The upper thermal boundary layer is more cooled because of the higher value of lattice thermal conductivity close to the surface. The decreasing lattice thermal conductivity increases the effective Rayleigh number, and then the convection becomes more active. As a result, the upper thermal boundary layer becomes thinner and its temperature gradient also becomes steeper.

The Fig. 52 (a) shows typical vertical profiles of thermal conductivity, temperature and viscosity around the upper thermal boundary layer. In the cases with the temperature-dependent thermal conductivity, the temperature gradient in the thermal boundary layer is not linear because the vertical heat flow must be kept constant in the layer. By considering the depth distribution of lattice thermal conductivity, the temperature gradient is more gentle close to the surface and is steeper around the bottom of the upper thermal boundary layer. Because the viscosity gradient also reflects the profile of temperature, the viscosity gradient is more gentle close to the surface and is steeper in the lowermost part of the upper thermal boundary layer. The diffusion terms of momentum include the terms propotional to the viscosity gradients, and then the convective flow is decayed rapidly across the layer with steeper viscosity gradients. In the lowermost part of
the upper thermal boundary layer with the lattice thermal conductivity, the pulls of the cold plumes are decayed because of the steeper viscosity gradient. Such a part promotes the lid to be separated from the underlying convective layer, and is observed as a sharp boundary between the lithosphere and the athenosphere (Gaherty, 2001). In addition, the small perturbation appears around the bottom of the upper thermal boundary layer, because the local Rayleigh number increases due to the steeper temperature gradient, the low viscosity and the low thermal conductivity. Therefore the upper thermal boundary layer becomes thinner than that in the model with constant thermal conductivity.

The weaker pulls by the smaller descending plumes are cut off, on the other hand, the stronger pulls by the larger downwellings are not cut off from the lid, by the strengthened diffusion in the lowermost part of the lid. In the upper part of the upper thermal boundary layer, the convective flows are not much diffused because of the more gentle viscosity gradient. As a result, the stronger pulls by the larger downwellings can affect on the surface deformation (Regenauer and Yuen, 1998). Such a situation seems to be valid for the Earth’s plate-tectonics, especially, the determination of the location for initiation of subduction.

Other planets, for example, Mercury, Venus and Mars, have deformations with various scales on their surfaces, and some of them are related to viscous flows inside the planets. Venus has mantle convection, which produces some kinds of tectonic features. However there is no plate tectonics on Venus, and therefore mantle flow must produce deformations of the surface by propagating
stresses across the thick lithosphere (Kaula, 1990; Kidder et al., 1996; Hansen et al., 1997; Roger et al., 1998). The lattice component of temperature-dependent thermal conductivity can promote the deformations of the surface by decreasing viscosity contrast near the surface of the lithosphere. The scales of the surface deformations may be dependnet on the scales of underlying viscous flows and the degree of the viscosity gradient in the lowermost part of the lithosphere. The deformations on the surface of Mars (Zuber, M.T., 2001) may be explained by the same arguments posed for Venus.

Not only mantle rocks but also ice has a strong temperature-dependent rheology (Hobbs, 1974; Echelmeyer et al., 1984). Some of the Galilean Satellites, for example, Europa, Ganymede and Callisto are covered by icy shells (Deschamps et al., 2001). Europa and Ganymede have very thick icy shells (some hundreds of kilometers) and many deformations with various scales have been found on their surfaces (McKinnon, 1997; McKinnon, 1999). Ice has also a rather strong temperature-dependence in the thermal conductivity (Hooke, 1998). Because the effects of lattice thermal conductivity on temperature-dependent rheology is more significant in the regions with lower temperature, it may promote much more readily deformations with smaller scales on icy surfaces of the Galilean Satellites (Schenk et al., 2001).
3.4.2 Influence of the Radiative Thermal Conductivity

I next investigated the influence of the temperature-dependent radiative thermal conductivity on thermal convection with the temperature-dependent viscosity. The thermal convection has higher temperature and faster convective flow, which are explained as discussed in section 2. The more heated fluid has smaller viscosity, which increases the effective Rayleigh number. The increasing effective Rayleigh number by decreasing viscosity overcomes the decreasing Rayleigh number by increasing thermal conductivity, because of the different orders of changes of them. Because the convective flow becomes more active, the upper thermal boundary layer becomes thinner and the temperature gradient also becomes steeper in the layer.

The significant influence of the radiative thermal conductivity is also found in the dynamics of the upper thermal boundary layer. The vertical profiles of thermal conductivity, temperature and viscosity are shown in Fig.52 (b). By considering the depth distribution of radiative thermal conductivity, the temperature gradient is steeper close to the surface and is more gentle around the bottom of the upper thermal boundary layer. Because the viscosity gradient also reflects the profile of temperature, the viscosity gradient is steeper close to the surface and is more gentle in the lower part of the upper thermal boundary layer. In the lower part of the upper thermal boundary layer with the radiative thermal conductivity, the lower part may be easily included in the underlain convective
layer, because of the more gentle viscosity gradient. Therefore, the upper thermal boundary layer becomes thinner than that in models with constant thermal conductivity. In the upper part of the upper thermal boundary layer, the convective flows are much diffused because of the steeper viscosity gradient. The pulls of cold plumes are diffused rapidly close to the surface and cut off from the upper thermal boundary layer. As a result, the stagnant-lid appears along the surface, and then the surface topography does not reflect the convective patterns any more, for example, the positions of plumes. Such a situation seems to be not valid for the plate-dynamics on the planets. Because lower temperature seems to be valid for the planets’ surfaces, the contribution of the radiative thermal conductivity is weaker than that of the lattice thermal conductivity. However, the formulation for the temperature-dependence of the radiative thermal conductivity is more uncertain than that for the lattice thermal conductivity. Recently, Hofmeister proposes that the radiative thermal conductivity of the mantle rocks is strongly affected by the grain sizes of the rocks and the existence of water (Hofmeister, 2004). It is therefore difficult to discuss about the surface dynamics associated with radiative thermal conductivity on the real planets.
4 Summary

I studied the influence of the temperature-dependent thermal conductivity of the thermal convection with temperature-dependent viscosity by carrying out the two-dimensional numerical experiments. I employed the two-dimensional rectangular box heated below and filled with the Boussinesq fluid. The infinite Prandtl number is imposed on the fluid. The free-slip condition is imposed on the top and bottom boundaries and the reflective condition is imposed on the side boundaries. The temperature-dependent thermal conductivity produces the variations of the thermal conductivity with both large and small scales. The large scale variation of the thermal conductivity reflects the vertical profile of the horizontally averaged temperature, which seems to be important for the efficiency of the heat transfer of the thermal convection. On the other hand, the small scale variation of the thermal conductivity reflects the patterns of the convective flow, for example, the positions of the plumes, which seem to be important for the local deformations by the convective flows. To simplify the discussions, I separated the variable thermal conductivity into two components; the depth-dependent thermal conductivity and the temperature-dependent thermal conductivity. The former expresses the vertical variation with larger scale, and the latter expresses the vertical and the horizontal variations with smaller scales.
4.1 Influence of the Depth-Dependent Thermal Conductivity on Thermal Convection

First, I investigated the thermal convection with the depth-dependent thermal conductivity and constant viscosity. I adopted the two-layer model of thermal conductivity: the decreasing thermal conductivity model (decreasing $k$ model) and the increasing thermal conductivity model (increasing $k$ model). The whole layer is separated into the upper part and the lower part. In the upper part of both models, the thermal conductivity equals to one. In the lower part of the decreasing $k$ model, thermal conductivity equals to 0.5. On the other hand, in the lower part of the increasing $k$ model, thermal conductivity equals to 1.5. I focused the changes of averaged temperature, averaged stream function and Nusselt number caused by the variation of the depth of the boundary between those two parts. These quantities are important to express the efficiency of the heat transport of the thermal convection. The Rayleigh number estimated at the surface equals to $10^5$ and the aspect ratio of the convective cell is fixed at one.

The fall of the boundary in the decreasing $k$ model from the surface $z=0$ to $z=0.1$ causes the decrease of the averaged temperature and the increases of the averaged stream function and the Nusselt number. The fall of the boundary from $z=0.1$ to $z=0.9$ does not cause significant changes of those quantities. The fall of the boundary from $z=0.9$ to the bottom $z=1$ causes the decreases of the averaged temperature, the averaged stream function and Nusselt number. I
also found similar features in the increasing $k$ model. The fall of the boundary from the surface to $z=0.1$ causes the increase of the averaged temperature and the decreases of the averaged stream function and the Nusselt number. The fall of the boundary from $z=0.1$ to $z=0.9$ does not causes significant changes of those quantities. The fall of the boundary from $z=0.9$ to the bottom causes the increases of the averaged temperature, the averaged stream function and Nusselt number. The layers form the surface to $z=0.1$ and from $z=0.9$ to the bottom are the upper and the lower thermal boundary layer, respectively. Therefore, it is more valid that the depth-dependent thermal convectivity is translated into the three-layer model: the upper thermal boundary layer, the interior and the lower thermal boundary layer.

Following these results, I employed the three-layer model of thermal conductivity. In each numerical experiment, thermal conductivity values of two layers equals to one, and the other is varied from 0.2 to 1.8. As the thermal conductivity in the upper thermal boundary layer increases, the averaged temperature decreases, and the averaged stream function and the Nusselt number increase. On the other hand, all quantities increase as the thermal conductivity in the lower thermal boundary layer increases. The increasing thermal conductivity in the interior does not cause the change of the averaged temperature, but causes the more gentle decreases of the averaged stream function and the Nusselt number. The influence of the change of the thermal conductivity in the interior can be explained as the influence by the change of the effective Rayleigh number. I
also considered the change of the horizontal scale of the convective cell caused by the changes of the thermal conductivity in the thermal boundary layers. As the thermal conductivity in the thermal boundary layers are smaller than that in the interior, the horizontal scales of the convective cells are longer.

To understand those features associated with the changes of the thermal conductivity in the thermal boundary layers, the loop-model was considered. The loop-model is the one-dimensional and analytical thermal convection model mimicking the two-dimensional thermal convection model. This model includes the heat transport only across the top and bottom boundaries: the heating at the bottom and the cooling at the surface. Such a simple model promotes us to understand basic mechanisms driving the thermal convection. The loop-model reproduced the features found in the two-dimensional model. That is, the result obtained from the two-dimensional models can be explained by the physics included in the loop-model. The higher thermal conductivity in the thermal boundary layers promotes the fluid to be heated or cooled more strongly. Because of such strong heating or cooling, the fluid can get enough buoyancy while the fluid moves along the shorter horizontal boundaries. This is the reason why the higher thermal conductivity models predict the shorter horizontal scale of the thermal convection. In addition, such a large buoyancy emphasizes the convective flow and the heat transfer. On the other hand, the lower thermal conductivity in the thermal boundary layers prevents the fluid from being heated or cooled. Because of such weakened heating or cooling, the fluid needs longer
horizontal scale to get the enough buoyancy. This is the reason why the lower thermal conductivity models predict the longer horizontal scale of the thermal convection. Such a small buoyancy decays the convective flow and the heat transfer. The difference between the thermal conductivity for both thermal boundary layers determines the averaged temperature of the layer.

As demonstrated above, the vertical variations of the thermal conductivity give the significant influence to the efficiency of the heat transfer of the thermal convection. The efficiency of the heat transport is one of the most important problems of the planetary thermal histories and the processes of the planetary evolutions (Stevenson, 1983). Such problems have been studied very well by using the parametrized convection models (Christensen, 1985). Because the two- or three-layer models of the thermal conductivity employed in this paper are proper to be included into the parametrized models, the vertical variation of thermal conductivity should be included in the studies of thermal histories of the planets (Sharpe and Peltier, 1978).

4.2 Influence of Temperature-Dependent Thermal Conductivity on Thermal Convection with Temperature-Dependent Viscosity

Next, I investigated the thermal convection with the temperature-dependent viscosity and the temperature-dependent thermal conductivity. The features
of the thermal convection with the temperature-dependent viscosity and constant thermal conductivity are studied very well by many researchers. The prior works showed that three convective regimes dependent on the viscous contrasts appeared: the small viscosity contrast (SVC), the transitional (TR) and the stagnant-lid (ST) regimes (Solomatov, 1995). The SVC, ST and TR regimes have no lids, stiff-lids and less-stiff lids near the surface, respectively. As the viscosity contrast across the layer increases, the lids become stiffer. Kameyama and Ogawa (2000) proposed an adittional convective regime: the elongated-ST regime characterized by the convective structures with stiff-lids along the surfaces and longer horizontal scales. In their models, the viscosity is fixed at the minimum on the bottom boundary, and increases as temperature decreases. In this paper, however, the viscosity is fixed at the maximum on the surface boundary, and decreases as temperature increases.

I adopted two non-dimensional numbers to describe the model establishment: the Rayleigh number defined at the surface and the viscosity contrast across the layer. Before considering the influence of the temperature-dependent thermal conductivity on the thermal convection with the temperature-dependent viscosity, I examined the convective structures with temperature-dependent viscosity and constant thermal conductivity. The plots of the horizontally averaged surface velocity and the Nusselt number versus the viscosity contrast indicated four convective regimes for the thermal convection: the SVC, the TR-I, the TR-II and the ST regimes. These regimes can be characterized by the stiffness of the upper
thermal boundary layers. In this paper, I emphasized that stiffness of the lid is
dependent on the vertical gradient of the viscosity rather than the value of the
viscosity itself.

Because the lids for the TR-I regime are less stiffer than those for the TR-
II regime, the downwelling can pull the fluid near the surface in the model for
the TR-I regime. On the other hand, the lids for the TR-II regime are stiffer.
Therefore, the cold plume can pull the fluid not near the surface but in the lower
part of the upper thermal boundary layer in the model for the TR-II regime.
For the ST regime, the lids are stiffer than those of the TR-II regime, and then
the fluid only in the lowermost part of the upper thermal boundary layer can be
pulled by the descending flow. The ST regime is also characterized by the small
disturbances around the bottom of the upper thermal boundary layer. In the
models with longer horizontal scales, the elongated-ST regime was found. This
is the result as the strengthened pull of the cold plume developed due to the cold
surface with longer horizontal scale.

The contribution of the lattice thermal conductivity is included in the thermal
convection with the temperature-dependent viscosity. The deformations in the
upper thermal boundary layer are mainly affected by the temperature-dependent
lattice thermal conductivity because of the lower temperature there. The lattice
components of the thermal conductivity decreases as temperature increases. In
the upper thermal boundary layer, the lattice thermal conductivity is higher close
to the surface, and is lower around the bottom. The temperature gradient also
becomes not a linear but a curve in the thermal boundary layer, because heat flow is kept to be constant there: the temperature gradient is more gentle close to the surface, and steeper around the bottom of the layer.

The viscosity gradient reflects the temperature gradient, and has same features with the temperature gradient. The variation of the viscosity gradient changes the strength of the diffusive terms in eq. (31): stronger diffusion for increased viscosity gradient, and weakened diffusion for decreased viscosity gradient. Around the bottom of the upper thermal boundary layer, the viscosity gradient is steeper, and then the diffusion is stronger. Therefore, the deformations caused by the convective flow underlying below the thermal boundary layer decrease because of such a stronger diffusion. In the downwelling, however, the isotherms are pulled downward and then the temperature and viscosity gradients are more gentle. In addition, the viscosity gradient near the surface is also more gentle as shown above. As a result, the pull by the downwelling is not decayed and can deform the surface topography.

Finally, I investigated influence of radiative thermal conductivity on the thermal convection with the temperature-dependent viscosity. The radiative thermal conductivity also causes the interesting changes on the convective structures, especially, the surface topography. However, such models may not be valid for the real Earth, because radiative thermal conductivity is important only under high temperature condition, for example, near the D''layer corresponding to the lower thermal boundary layer in the Earth’s mantle. The radiative component
increases as temperature increases, which produces temperature and viscosity gradients with the opposite features to those for the lattice components: the viscosity gradient is steeper close to the surface, and more gentle around the bottom of the upper thermal boundary layer. The lower part is more entrained into the underlying convective layer due to the more gentle viscosity gradient. On the other hand, temperature close to the surface is lower, resulting in higher viscosity close to the surface. The viscosity gradient is also steeper close to the surface. Because of the combination of the higher viscosity and the steeper viscosity gradient close to the surface, the more stagnant-lid appears along the surface.

As illustrated above, the structures of the thermal convection, especially, the deformations in the upper thermal boundary layer can be affected by the temperature-dependent thermal conductivity. Such features are consistent with the surface topographies and the dynamics in the lithosphere on the planets and the satellites (Solomatov and Moresi, 1996).

It is known that the thermal conductivity of the mantle rocks is dependent on temperature and pressure, and some models of the thermal conductivity for the whole mantle have proposed (e.g., Hofmeister, 1999). Because there are uncertainties in the observations of the values of the thermal conductivity of the mantle rocks and minerals, the temperature-dependence of the thermal conductivity is not accurate as compared to the temperature-dependence of the viscosity. Therefore, it is not easy to accurately estimate the influence of the variable thermal conductivity of the mantle rocks on the mantle convection. However, the
basic physics discussed in this paper; i.e., the variable thermal conductivity of the mantle rocks has significant roles on the mantle convection and it related deformations in the lithosphere, are very important for future studies.
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Table 1: The list of numerical experiments presented in this paper. The Rayleigh number estimated at the surface $Ra_s$ is fixed at $10^2$. The aspect ratio of the box is fixed at 1. Values of $\beta$ in eq.(33) and common logarithm of viscosity contrast across the layer $\gamma_\nu$ and names of the models are propted.
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Table 2: The list of numerical experiments presented in this paper. The Rayleigh number estimated at the surface $Ra_s$ is fixed at $10^3$. The aspect ratio of the box is fixed at 1. Values of $\beta$ in eq.(33) and common logarithm of viscosity contrast across the layer $\gamma_\nu$ and names of the models are protted.
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14 & 6.08 & C-C14 & C-L14 & C-R14 \\
13 & 5.64 & none & none & C-R13 \\
12 & 5.21 & C-C12 & C-L12 & C-R12 \\
11 & 4.77 & C-C11 & C-L11 & C-R11 \\
10 & 4.34 & C-C10 & C-L10 & C-R10 \\
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8 & 3.47 & C-C08 & C-L08 & C-R08 \\
7 & 3.04 & C-C07 & C-L07 & C-R07 \\
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5 & 2.17 & C-C05 & C-L05 & none \\
4 & 1.73 & C-C04 & none & none \\
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\caption{The list of numerical experiments presented in this paper. The Rayleigh number estimated at the surface $Ra_s$ is fixed at $10^2$. The aspect ratio of the box is fixed at 2. Values of $\beta$ in eq. (33) and common logarithm of viscosity contrast across the layer $\gamma_\nu$ and names of the models are propted.}
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